Progression in written calculations

The 2014 National Curriculum

Aims

The National Curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

Calculators should not be used as a substitute for good written and mental arithmetic. They should therefore only be introduced near the end of Key Stage 2 to support pupils' conceptual understanding and exploration of more complex number problems if written and mental arithmetic are secure.

Key Stage 1	Lower Key Stage 2	Upper Key Stage 2
The principal focus of mathematics teaching in Key Stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources (e.g. concrete objects and measuring tools). By the end of Year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.	The principal focus of mathematics teaching in lower Key Stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers. By the end of Year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.	At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation.

It is important that children understand the concepts of the four operations:

ADDITION	SUBTRACTION:	MULTIPLICATION	DIVISION
• Combining two or more groups to give a total or sum • Increasing an amount They also need to understand and work with certain principles, i.e. that it is: • the inverse of subtraction • commutative i.e. 5 + 3 = 3 + 5 • associative i.e. 5 + 3 + 7 = 5 + (3 + 7)	• Removal of an amount from a larger group (take away) • Comparison of two amounts (difference) They also need to understand and work with certain principles, i.e. that it is: the inverse of addition • not commutative i.e. 5 - 3 is not the same as 3 - 5 • not associative i.e. 10 - 3 - 2 is not the same as 10 - (3 – 2)	• Repeated addition They should also be familiar with the fact that it can be represented as an array. They also need to understand and work with certain principles, i.e. that it is: • the inverse of division • commutative i.e. 5 x 3 is the same as 3 x 5 associative i.e. 2 x 3 x 5 is the same as 2 x (3 x 5)	• Repeated subtraction • Sharing into equal amounts • Grouping They also need to understand and work with certain principles, i.e. that it is: • the inverse of multiplication • not commutative i.e. 15 ÷3 is not the same as 3 ÷ 15 not associative i.e. 30 ÷ (5 ÷ 2) is not the same as (30 ÷ 5) ÷

Language should be consistent across the whole school:

- It is school policy that we will use the terms hundreds, tens and ones, (rather than units).
- The word 'regrouping' will be used rather than 'carrying', 'exchanging' or 'borrowing'

Children should be encouraged to ask themselves:

- **Can I do it in my head using a mental strategy?**
- **Could I use some jottings?**
- **Should I use a written method?**

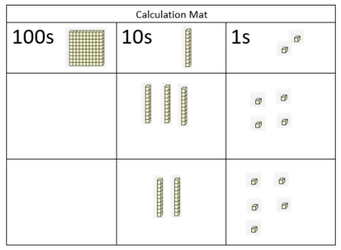
PROGRESSION IN WRITTEN METHODS FOR ADDITION

EARLY YEARS	YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5	YEAR 6
Early Learning Goal Using quantities and objects, children add and subtract two single-digit numbers and count on or back to find the answer.	• Add and subtract one-digit and two digit numbers up to 20 • Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems. Children should use a variety of concrete apparatus and pictorial representations to ensure they understand and can explain their thinking and should begin to record their calculations.	• solve problems with addition and subtraction using concrete objects and pictorial representations, including those involving numbers, quantities and measures. • apply increasing knowledge of mental and written methods. • add and subtract numbers using concrete objects, pictorial representations, and mentally, including: – a two-digit number and ones, a two-digit number and tens , two two-digit numbers – adding three one-digit numbers • show that the addition of two numbers can be done in any order (commutative)	• add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. Use base ten and the calculation mat to support column addition.	• Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. Use base ten and the calculation mat to support column addition	• Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) Use place value counters to support adding decimals	• *Pupils practise addition of larger numbers, using the efficient written methods of columnar addition and subtraction*
Combining two parts to make a whole Starting at the bigger number and counting on- using cubes Those who are ready may record their own calculations. **Concrete addition (using counters, cubes etc)** 2 + 1 = 3 **Pictorial addition** 2 + 1 = 3	Begin to record calculations eg 13 + 4 = 17 Use a numberline to count on from the first number then from the largest number: Regrouping to make 10 using ten frame. 8 + 6 = Be able to solve missing number problems and prove their answer is correct using practical equipment – eg 3 + ☐ = 7	Use of base 10 to combine two numbers. 41 + 8 36 + 25 10s 1s 6 1 Children should be encouraged to regroup 10 ones as 1 ten. This is the start of children understanding regrouping in vertical addition. Expanded addition – add the least significant digits first	Expanded method 67 + 56 67 + 56 = 123 (60 + 7) + (50 + 6) 60 + 50 = 110 7 + 6 = 13 110 + 13 = 123 Leading to column addition 435 + 213 = 648 435 + 217 = 652 435 + 287 = 722 Also include: 679 + 73 = 752 251 + 73 = 334	Column addition 3251 + 5413 = 8664 3251 + 5473 = 8634 2938 + 5423 = 8361 8958 + 5873 = 14731 Also include: 3758 + 413 = 4171 3778 + 483 = 4261 351 + 234 + 423 = 1008 355 + 234 + 473 = 1062 **Decimal addition in the context of money.**	Column addition – various regrouping 37234 + 75479 = 112713 **Decimal addition in the context of money** Also include: 2318 + 53 + 925 = 3296 23.14 + 560.83 + 46.71 = 630.68 **Decimal addition in the context of money and measures to 3 d.p**	<u>Formal written method</u> 76259 + 68068 + 7514 = 151841 **Numbers with different decimal places** 5.234 + 43.19 + 387.3 5.234 + 43.190 + 387.300 = 435.724

USE OF THE CALCULATION MAT FOR COLUMN ADDITION

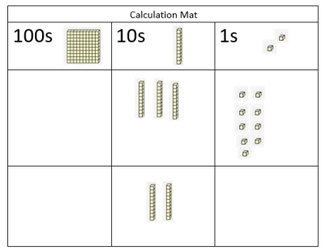
Using the calculation mat for column addition – no regrouping

Make both numbers using base 10 and place them on the calculation mat. The ones should be arranged in twos so that children can see how many there are without needing to count them all individually.
At this point get the children to re-write the calculation as column addition so that it mirrors the calculation mat.



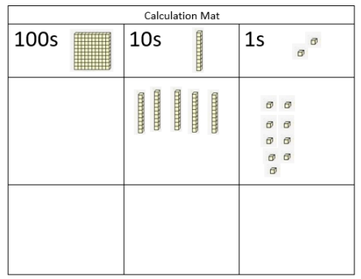
34
+ 25

Then 'push up' the ones.



34
+ 25
9

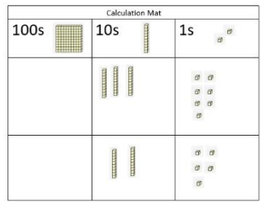
Then 'push up' the tens, ensuring that the children count 10, 20, 30, 40, 50 not 1, 2, 3, 4, 5.



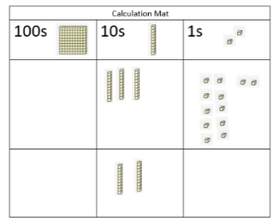
34
+ 25
59

Using the calculation mat for column addition – regrouping

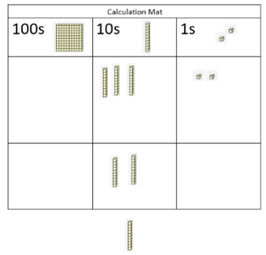
Make both numbers with base 10 and place them on the calculation mat. Then write the calculation, ensuring the digits are aligned correctly and it mirrors the calculation mat.



When the children 'push up' the ones should notice that there are 12 and so they need to regroup 10 ones as 1 ten.

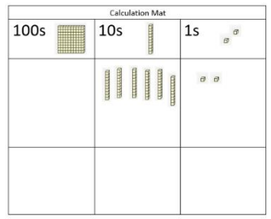


Replace the regrouped ten at the bottom of the tens column so that it exactly mirrors the calculation mat.



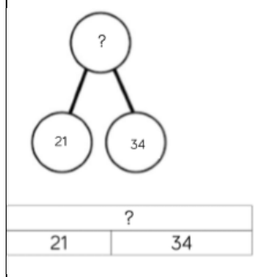
3 7
+ 2 5
2
1

Push up the tens to complete the answer.



3 7
+ 2 5
6 2
1

CONCEPTUAL VARIATION: DIFFERENT WAYS TO ASK CHILDREN TO SOLVE 21 + 34 = 55



Word problems:
In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

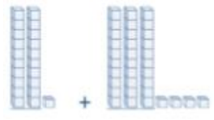
21 + 34 = 55. Prove it

21
+34
___

21 + 34 =

☐ = 21 + 34

Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

10s	1s
🟡🟡	1
🟡🟡🟡	?
?	5

Across Key Stage 2, provide plenty of opportunities to use and apply written methods in a range of contexts.

Use five of these numbers to make the calculation correct
4, 4, 4, 9, 9, 9

☐☐☐
+ ☐☐
5 4 8

What is the mistake?

1 2 . 3
+ 9 . 8
2 1 . 1 1

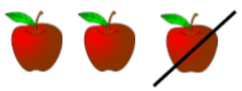
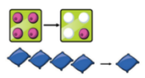
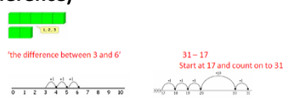
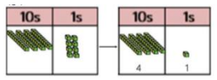
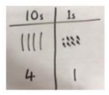
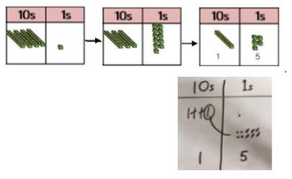
What is the missing number?

5 4 8
+ 7 ☐ 7
1 3 2 5

- Find two 3-digit numbers with a sum of 465.
- Beth has made a necklace with 123 pink beads and 238 purple beads. How many beads are on the necklace altogether?

Find the different totals you can make by using any three of these numbers:
1.07, 0.3, 37.03, 17.73, 31.7

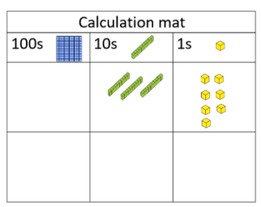
PROGRESSION IN WRITTEN METHODS FOR SUBTRACTION

EARLY YEARS	YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5	YEAR 6
Early Learning Goal Using quantities and objects, children add and subtract two single-digit numbers and count on or back to find the answer.	• Add and subtract one-digit and two digit numbers up to 20 • Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems. Children should use a variety of concrete apparatus and pictorial representations to ensure they understand and can explain their thinking and should begin to record their calculations.	• solve problems with addition and subtraction using concrete objects and pictorial representations, including those involving numbers, quantities and measures. • apply increasing knowledge of mental and written methods. • add and subtract numbers using concrete objects, pictorial representations, and mentally, including: – a two-digit number and ones, a two-digit number and tens , two two-digit numbers – adding three one-digit numbers • show that the addition of two numbers can be done in any order (commutative)	• add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. Use the calculation mat to support column subtraction.	• Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. Use the calculation mat to support column subtraction.	• Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	• *Pupils practise subtraction … for larger numbers, using the efficient written methods of columnar addition and subtraction*
Taking away ones **Physically take away and remove objects from a whole.** **Those who are ready** may record their own calculations.  3 – 1 = 2  **7 – 3 = 4**	**Begin to record calculations:** Eg 17 – 6 = **Physically take away and remove objects from a whole.** ten frames, cubes and other items  Draw the concrete resources and cross out the correct amount.  **Counting back** Use number lines or number tracks to count back from the first number.  Encourage children to use an empty number line .   **Counting on (finding the difference)** 	**Use a numberline to find the difference by counting on.** **Use base ten to subtract numbers.** 48 - 7  Represent the base 10 pictorially.  **Regrouping: 41 - 26 =**  When children are secure and confident with subtracting 1 and 2-digit numbers using the strategies outlined in the maths policy, they can progress to using written methods starting with calculation mats and base 10 resources.	**Expanded method** to enable conceptual understanding using two then three-digit numbers 745 - 219 700 + 40 + 5 - 200 + 10 + 9 500 + 20 + 6 = 526 leading to **column subtraction** 5 3 6 - 3 2 1 2 1 5 7 8 4 - 2 3 7 5 4 7 7 3 5 - 2 7 8 4 5 7 6 0 3 - 2 4 7 3 5 6 **Include zero** 6 0 3 - 2 4 7 3 5 6 **Also include:** 5 8 3 - 3 2 5 5 1 4 7 4 - 8 5 3 8 9	**Column subtraction** 5 8 3 7 - 1 3 2 4 4 5 1 3 4 6 7 - 2 3 9 2 2 3 7 5 7 5 5 3 - 3 7 2 8 3 8 2 5 6 3 2 5 - 1 5 3 6 4 7 8 9 **Include zero** 6 0 4 3 - 4 7 8 1 1 2 6 2 **Also include:** 1 5 3 4 - 2 5 4 1 2 8 0 3 1 5 5 - 5 3 6 2 6 1 9 **Decimal subtraction in the context of money.**	**Column subtraction - various regrouping** 7 5 3 6 5 - 3 2 5 3 9 4 2 8 2 6 **Also include:** 7 5 3 6 6 - 6 2 7 7 4 7 3 9 2 0 4 3 9 - 5 2 4 7 1 5 1 9 2 **Decimal subtraction in the context of money and measures to 3 d.p**	**Column method** Include numbers with different decimal places 327.5 - 62.63 3 2 7 . 5 0 - 6 2 . 6 3 2 6 4 . 8 7 645.27 - 351.8 3 2 7 . 5 0 - 6 2 . 6 3 2 6 4 . 8 7 **Decimal subtraction in the context of money and measures to 3 d.p**

USE OF THE CALCULATION MAT FOR COLUMN SUBTRACTION

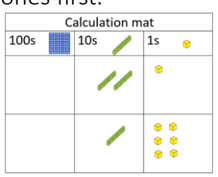
Using the calculation mat for column subtraction – no regrouping

Make the first numbers using base 10 and place it on the calculation mat. Discuss why we only need to make the first number. Then get the children to re-write the calculation as column subtraction so that it mirrors the calculation mat.



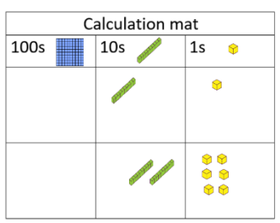
37
-16

This time we 'push down' the ones first.



37
-16
1

Then 'push down' the tens. By leaving the ones and tens you have subtracted on the bottom section of the calculation mat, the children can check their answer is correct by doing the inverse operation.

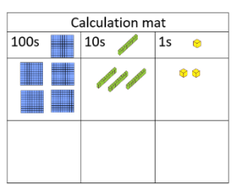


37
-16
21

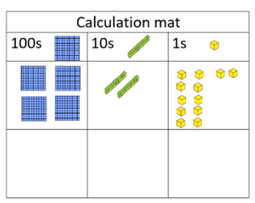
Using the calculation mat for column subtraction – regrouping

Write the calculation and then make the first number using base ten and the calculation mat.

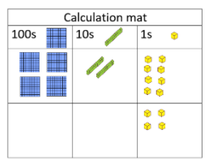
432
-124



As you cannot subtract 4 from 2, you need to regroup a ten into ten ones.



Then 'push down' the ones.



$4^{2}\cancel{3}^{1}2$
-124
8

Then 'push down' the tens,

$4^{2}\cancel{3}^{1}2$
-124
08

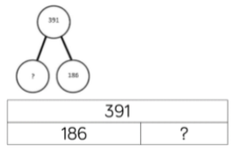
then the hundreds.

$4^{2}\cancel{3}^{1}2$
-124
308

432
-124

Check the correct value has been taken away.

CONCEPTUAL VARIATION: DIFFERENT WAYS TO ASK CHILDREN TO SOLVE 391 - 186



Raj spent £391, Timmy spent £186. How much more did Raj spend?

Calculate the difference between 391 and 186.

□ = 391 – 186

391
-186
___

What is 186 less than 391?

Missing digit calculations

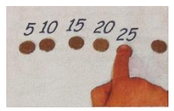
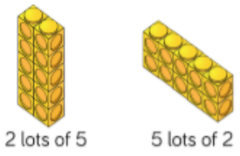
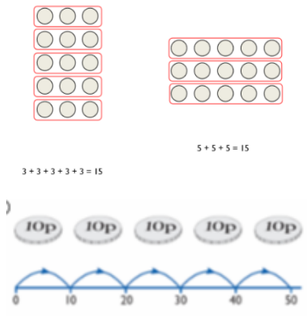
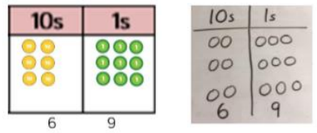
Across Key Stage 2, provide plenty of opportunities to use and apply written methods in a range of contexts.

What are the missing digits?

Use the digits 1, 2, 3, 4, 6, 9 to make the calculation correct

- The Smith family has saved £675 towards their summer holiday. The cost of the holiday is £2019. How much more do they need to save?
- At the beginning of a cricket match there were 742 people watching. At tea-time 218 people went home. How many were left?
- Two numbers have a difference of 1.58. One of the numbers is 4.72. What is the other? Is this the only answer?
- Gordon won £363 630 on the lottery and Betty won £4387, how much more did Gordon win?
- After a sale, Boots cost £55.23 and trainers cost £34.78. How much less do the trainers cost?

PROGRESSION IN WRITTEN METHODS FOR MULTIPLICATION

EARLY YEARS	YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5	YEAR 6
Early Learning Goal Children solve problems, including doubling.	• Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher • *Make connections between arrays, number patterns and counting in twos, fives and tens.* Children should use a variety of concrete apparatus and pictorial representations to ensure they understand and can explain their thinking and should begin to record their calculations.	• Recall and use multiplication facts for the 2, 5 and 10 multiplication tables. • Calculate mathematical statements for multiplication and division within the multiplication table and write them using the multiplication (x), division (÷) and equals (=) signs. • Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot. • Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in contexts.	• Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including two-digit by one-digit numbers, using mental and progressing to formal written methods. • Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. Use place value counters to support multiplication	• Multiply two-digit and three-digit numbers by a one-digit number using formal written layout. • *Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers.* Use place value counters to support multiplication	• Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.	• Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication. • Multiply one-digit numbers with up to two decimal places by whole numbers.
Recognising and making equal groups. Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.  **Doubling** They may develop ways of recording calculations using pictures, etc. A child's jotting showing double three as three cookies on each plate. 	Counting in multiples, use cubes, and other objects in the classroom   **Arrays- showing commutative multiplication** **2 × 5 = 5 × 2**  	**Continue to solve simple one step problems involving multiplication calculating the answer using concrete objects, pictorials and arrays.**  10 + 10 + 10 + 10 + 10 = 50 10 X 5 = 50	**Use place value counters, (base 10 can also be used), and move onto representing pictorially.** 23 x 3  Grid method 23 x 3 × | 20 3 3 | 60 9 = 69 16 x 4 × | 10 6 4 | 40 24 = 64 32 x 8 × | 30 2 8 | 240 16 = 256	**Grid method** 135 x 6 × | 100 30 5 6 | 600 180 30 = 810 **Expanded method to enable conceptual understanding** 2 4 × 3 1 2 6 0 7 2 **Leading quickly to formal written method** 4 2 × 3 = 1 2 6 3 6 × 4 = 1 4 4 (2) 3 1 2 × 6 = 1 8 7 2 (1) 2 7 3 × 7 = 1 9 1 1 (5 2)	**Formal written method** 2 5 1 3 × 7 = 1 7 5 9 1 (3 2) 6 5 7 9 × 8 = 5 2 6 3 2 (4 6 7) **Long Multiplication** 2 7 × 3 4 = 1 0 8 / 8 1 0 / 9 1 8 (2 3) 7 8 × 6 4 = 3 1 2 / 4 6 8 0 / 4 9 9 2 (3 4) 1 2 4 × 2 6 = 7 4 4 / 2 4 8 0 / 3 2 2 4 (1 2 / 1 1) 2 3 7 4 × 3 2 = 4 7 4 8 / 7 1 2 2 0 / 7 4 9 6 8 (1 2 1 / 1)	**Formal written method** 6 0 2 7 × 3 4 = 2 4 1 0 8 / 1 8 0 8 1 0 / 2 0 4 9 1 8 (1 2 / 2 2 / 1) 4 3 7 8 × 7 3 = 1 3 1 3 4 / 3 0 6 4 6 0 / 3 1 9 5 9 4 (1 2 2 / 2 2 2) 8.7 × 6.0 = 52.2 (4) 8.68 × 7.00 = 60.76 (4 5) **Also include** 784.9 × 6.0 = 4909.4 (5 2 5) 41.68 × 7.00 = 291.76 (1 4 5) **Decimal multiplication in the context of money and measures** 4 7.3 × 6 2.0 = 9 4.6 / 2 8 3 8.0 / 2 9 3 2.6 (1 / 1 1) 3 1.5 6 × 2 3.0 0 = 9 4.6 8 / 6 3 1.2 0 / 7 2 5.8 8 (1 / 1)

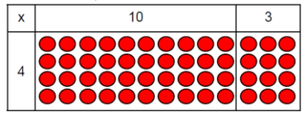
USE OF THE BASE TEN AND PLACE VALUE COUNTERS FOR MULTIPLICATION

Grid Method

4 x 13 =

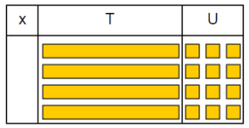
Show the link with arrays to first introduce the grid method.

4 rows of 10
4 rows of 3



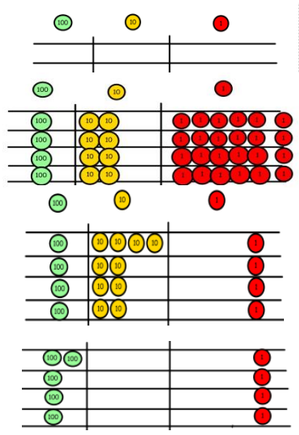
Move quickly on to using Base 10, or T/O place value counters to move towards a more efficient method.

4 rows of 13



4 x 126 =

Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



Calculations
4 x 126

Fill each row with 126.

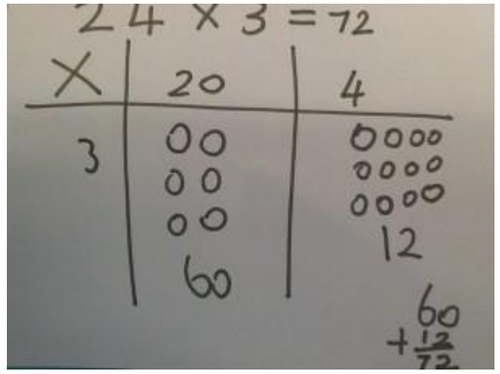
Calculations
4 x 126

Add up each column, starting with the ones, regrouping where needed.

4 x 126 = 504

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Children can continue to be supported by place value counters when they move onto the column method for multiplication.

It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

CONCEPTUAL VARIATION: DIFFERENT WAYS TO ASK CHILDREN TO SOLVE 6 X 23

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

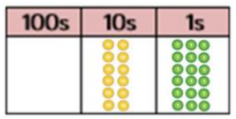
Find the product of 6 and 23

6 × 23 =

☐ = 6 × 23

$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ 6 \\ \hline \end{array}$$

What is the calculation?
What is the product?



Across Key Stage 2, provide plenty of opportunities to use and apply written methods in a range of contexts.

- There is space in the car park for 17 rows of 32 cars. How many cars can park?
- How many hours are there in one year?
- What is the total mass of 235 screws each weighing 6g?
- Find the area of a swimming pool which is 25m long and 7.5m wide

- I buy 1.6 kg of apples. They cost 65p per kg. how much do I pay?
- An exercise book is 15mm thick. How thick will a pile of 5 exercise books be?
- How many different answers can be made by using the digits 2, 3 and 4 in this calculation? ☐ ☐ x ☐ =

- Organise the digits 9, 7, 5 and 3 into this calculation to give the greatest possible product ☐.☐ ☐ x ☐
- Which is closer to 100: 5.2 × 17 or 7.2 × 15? Use written methods to prove your answer
- Abbie says that 23.4 × 5 will be bigger than 53.4 × 2. Is she correct?
- A can of drink contains 0.33 litres. How many litres are in 15 cans?

$$\begin{array}{r} \square 4 \square \\ \times \qquad 6 \\ \hline 2052 \end{array}$$

PROGRESSION IN WRITTEN METHODS FOR DIVISION

EARLY YEARS

Early Learning Goal
Children solve problems, including halving and sharing.

YEAR 1

- Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- *Make connections between arrays, number patterns and counting in twos, fives and tens.*

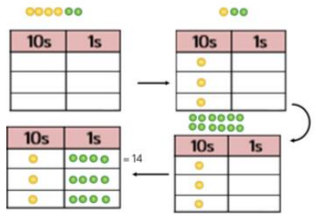
Children should use a variety of concrete apparatus and pictorial representations to ensure they understand and can explain their thinking and should begin to record their calculations.

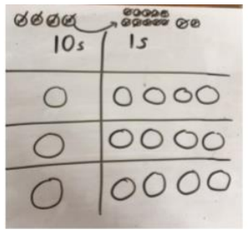
YEAR 2

- Recall and use multiplication facts for the 2, 5 and 10 multiplication tables.
- Calculate mathematical statements for multiplication and division within the multiplication table and write them using the multiplication (x), division (÷) and equals (=) signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in contexts.

Sharing using place value counters

42 ÷ 3 =





YEAR 3

- *Develop reliable written methods for division, starting with calculations of two-digit by one-digit numbers and progressing to the formal written methods of short division.*

Use place value counters to support partitioning

Partitioning

39 ÷ 3

$$3\overline{)30+9} = 10+3 = 13$$

64 ÷ 4

$$4\overline{)40+24} = 10+6 = 16$$

72 ÷ 3

$$3\overline{)60+12} = 20+4 = 24$$

YEAR 4

- *Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers.*

Use place value counters to support partitioning

Partitioning

119 ÷ 7

$$7\overline{)70+49} = 10+7 = 17$$

216 ÷ 9

$$9\overline{)180+36} = 20+4 = 24$$

Short Division

$$3\overline{)63} = 21$$

$$6\overline{)84} = 14$$

YEAR 5

- Divide numbers up to 4 digits by a one- or two-digit number using a formal written method of short division and interpret remainders appropriately for the context.

Short division

$$7\overline{)161} = 23 \qquad 8\overline{)2732} = 34$$

$$6\overline{)14466} = 241 \qquad 7\overline{)9243135} = 1345$$

There are 421 children here today. How many teams of 9 can we make?

$$9\overline{)4201} = 46 \text{ r } 7 \text{ teams}$$

206 tickets were sold for a concert; there are 7 seats per row, how many rows are needed?

$$7\overline{)2066} = 29 \text{ r } 3 \text{ rows}$$

496 ÷ 11 becomes

$$11\overline{)496} = 45 \text{ r } 1$$

Answer: $45\frac{1}{11}$

YEAR 6

- Divide numbers up to 4 digits by a two-digit number using the formal written method of short and long division where appropriate, interpreting remainders according to the context.

Formal written method

$$11\overline{)4955} = 45 \qquad 21\overline{)5084} = 24$$

$$12\overline{)4373112} = 361$$

$$26\overline{)3432} = 132$$

$$-2600 \quad \times 100$$

$$832$$

$$-780 \quad \times 30$$

$$52$$

$$-52 \quad \times 2$$

$$0$$

$$15\overline{)3960.0} = 26.4$$

$$-300 \quad \times 20$$

$$96$$

$$-90 \quad \times 6$$

$$6.0$$

$$-6.0 \quad \times 0.4$$

$$0$$

432 ÷ 15 becomes

$$15\overline{)4320} = 28 \text{ r } 12$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$15\overline{)4320} = 28$$

$$-300 \quad 15\times 20$$

$$132$$

$$-120 \quad 15\times 8$$

$$12$$

$\frac{12}{15} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

$$15\overline{)4320.0} = 28.8$$

Answer: 28·8

Also include

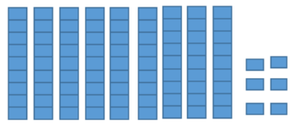
$$6\overline{)3443.1536} = 57.26$$

USING BASE TEN OR PLACE VALUE COUNTERS FOR DIVISION

96 ÷ 6 =

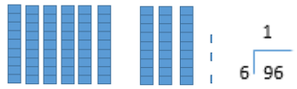
Show how this can also be written like this:

Make 96 with the base-10 apparatus or place value counters.



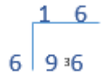
Reinforce that this is the only operation that we start with the largest value - *90*

How many groups of 6 ten sticks can we make with these 9 ten sticks? You can make 1 group of 6 tens and there will be 3 tens left over.
Record this above the tens digit.

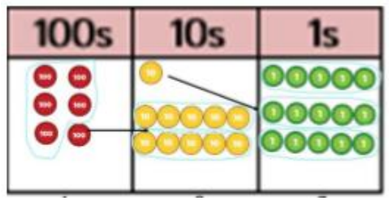


Then regroup the 3 tens across, so that we now have 36. How many groups of 6 are there in 36?
<u>The children may at this stage need to regroup the 3 tens in to 30 ones</u> to be able to find out how many groups of 6 there are in 36.

Then record the answer above the ones digit.



615 ÷ 5

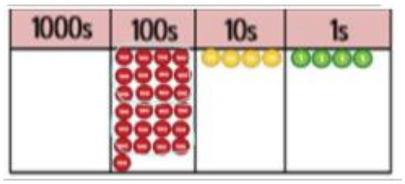


1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Regroup 1 hundred into 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Regroup 1 ten into 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

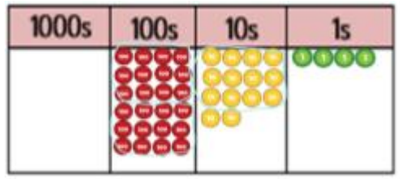
Long division using place value counters

2544 ÷ 12

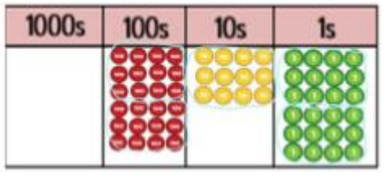
We can't group 2 thousands into groups of 12 so need to regroup them.



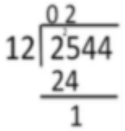
We can group 24 hundreds into two groups of 12 and are then left with 1 hundred.

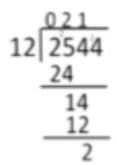


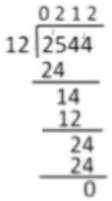
After regrouping the hundred, we have 14 tens. We can make 1 group of 12 tens, which leaves 2 tens.



After regrouping the 2 tens, we have 24 ones. We can group them into 12 groups of 2, which leaves no remainder.





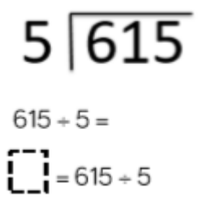


CONCEPTUAL VARIATION: DIFFERENT WAYS TO ASK CHILDREN TO SOLVE 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?

I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many pupils will be in each group?



What is the calculation?
What is the answer?

Across Key Stage 2, provide plenty of opportunities to use and apply written methods in a range of contexts.

- Work out whether or not 29 is a factor of 811
- How many 35p packets of stickers can I buy with £5?
- Coaches have 56 seats for passengers. How many coaches are needed to take 275 people on a trip?

- Pencils come in packs of 12. How many packs does a school need to buy to get 310 pencils?
- My mobile phone costs 18p per minute for national calls. If I put £5 on my card, how many minutes can I talk for?
  The area of a rectangular games hall is 384 square metres. If the length is 24 metres, how wide is it?

- Work out the missing digit for 37□ ÷ 17, when there is a remainder of 5.
- I bought some pencils that cost 15p each. I paid £5.85. How many pencils did I buy?
- Four children collected £19 for charity. They each collected the same amount. How much was this?

- Rupert saves the same amount of money each month. He saved £149.40 in a year. How much money does he save each month?
- Three bags of crisps weigh 130.5g, how much does one bag weigh?