Ernesford Grange Primary School

Teaching Calculations

(Revised March 2024 to ensure consistency with vocabulary listed in **Sentence Stems / Key Learning** document.)

Teaching Calculations at Ernesford Grange Primary School

INTRODUCTION

This policy is written in line with the requirements of the 2014 Primary National Curriculum and the White Rose schemes of learning, which is used for our long-term planning. It ensures there is consistency and progression across the school and sets out the expectations for each year group in terms of strategies for calculations which will be taught, explored and applied to a range of contexts.

Concrete, Pictorial, Abstract

The children's understanding of these calculation strategies will be underpinned by a secure understanding of place value. At EGPS we teach using a CPA (concrete, pictorial, abstract) approach. Understanding in all areas of maths will be developed by children using concrete resources to expose the different structures, and interpreting and using pictorial representations before moving onto solve abstract calculations. There are a combination of concrete and pictorial representations which will be met regularly though the children's maths learning journey. The CPA process/approach for the current maths focus will be clearly exemplified on maths working walls in each year group.

Checking Children's Understanding

Teachers will be aware, not only of their year group's expectations but those the children have learnt previously. It is important that teachers check the children's understanding of the previous teaching before moving on, in order for the children to really master the mathematics curriculum. Where individuals or groups of children do not show a secure understanding of what has been taught previously, it will be necessary to track back, either through whole-class teaching or individual or group intervention.

Mental Strategies

This document also contains essential information about the mental strategies that children will be taught. It is important that these are given high-priority during maths teaching and practised regularly, so that children develop the mental skills which are required across all areas of maths.

Fluency

Fluency with rapid recall of basic facts and use of efficient calculation methods is achieved through the use of NCETM Mastering Number sessions in EY and KS1 and discrete fluency sessions in KS2. Children are encouraged from an early age to subitise and use knowledge to calculate, rather than relying solely on counting.

.

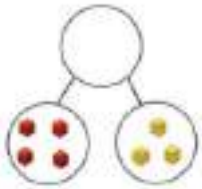
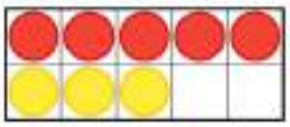
ADDITION

Early Years

Objectives from Early Years Foundation Stage Framework: Number	Objectives from Early Years Foundation Stage Framework: Numerical Patterns	Key Vocabulary
• Have a deep understanding of number to 10, including the composition of each number. • Subitise (recognise quantities without counting) up to 5. • Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.	• Verbally count beyond 20, recognising the pattern of the counting system. • Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity. • Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	More than, less than Fewer, most, least, same Add, put together Equal ones

Children should:

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts. They should explore number and the different ways the numbers can be partitioned (representing and using number bonds within 10).
- Experience subitising within 5 in a range of different contexts.
- Relate the counting sequence to cardinality, seeing that the last number spoken gives the number in the entire set.
- Understand that all numbers can be made of 1s.
- Explore the composition of numbers up to 10.
- Count to 20 and beyond from a range of starting points.


ADDITION
Year 1

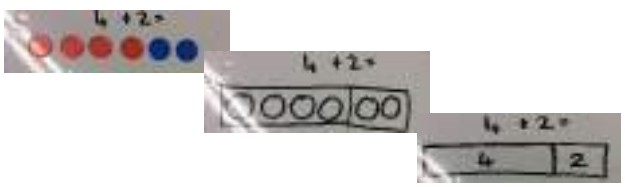
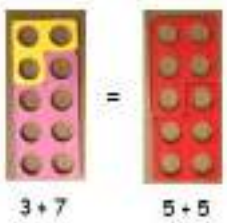
National Curriculum Objectives: Addition objectives from Addition and Subtraction Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Read, write and interpret mathematical symbols involving addition (+) and equals (=) signs. • Represent and use number bonds within 20. • Add one-digit and two-digit numbers to 20, including 0. • Solve one step problems that involve addition, using concrete objects and pictorial representations and missing number problems.	• Count forwards to and across 100, beginning with 0 or 1, or from any given number. • Count, read and write numbers to 100 in numerals. • Given a number, identify one more. • Identify and represent numbers using objects and pictorial representations including the number line. • Read and write numbers from 1-20 in numerals and words.	Addition, add, altogether, put together, sum, and, plus, total, combine Part, whole Equals, equal to, same as More than, less than, fewer, most, least Tens, ones Addend Partition

Children should:

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts. They should explore number and the different ways the numbers can be partitioned (representing and using number bonds within 20). Children should use abstract numbers to show each group.



- Read and write the addition (+) and equals (=) signs within number sentences. Specific learning should take place, through exploration of number and use of practical resources, around the equals sign - ensuring children understand its role within a number sentence and that it does not always just indicate where the answer goes.

- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them: 8 + 3 = ☐ , ☐ = 15 + 4 and 14 = ☐ + 9.

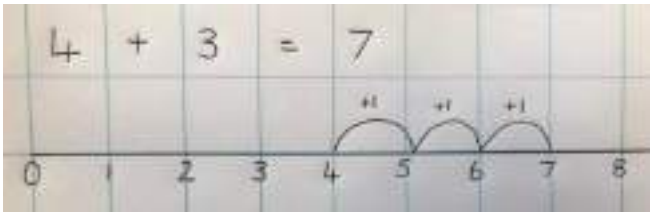
- Use the language of parts and wholes, (which can be represented by objects on the ten frame), as an introduction to **aggregation** (where parts are combined to make a whole) and **partitioning** (where the whole is split into parts.)
- Where children are not yet secure in their Early Learning Goals, they should be given the opportunity to consolidate these foundations.

Mental Methods: The children should be taught to use the following mental strategies, and to use jottings to support their methods:

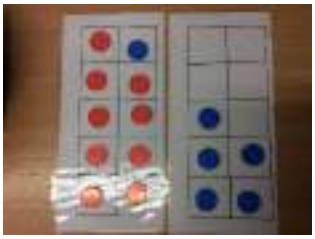
- Counting on in ones
- Re-ordering the numbers when adding e.g. putting the larger number first
- Counting on and back in ones, twos and tens
- "Think 10" - Looking for pairs of numbers that equal 10
- Partitioning small numbers to bridge tens e.g. 8+3 = 8+2+1
- Partitioning using known facts e.g. double and adjust 5+6 = 5+5+1
- Adding 9 to a number by adding 10 and then subtracting 1
- Recalling number bonds to 10 and 20 in several different forms (e.g. 9+7 = 16, 16-7=9 and 7=16-9)

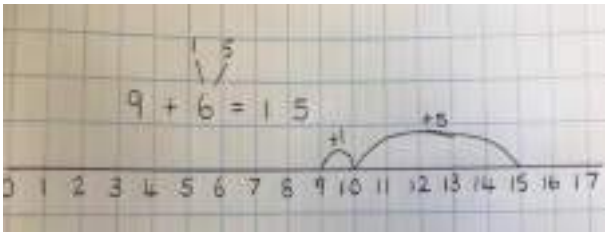
Written Methods: Building on the prior learning and exploration of number outlined above, children should:

Step 1 - Use numbered number lines to add, by counting on in ones or in small jumps using mental strategies to support.



Step 2 - Once confident using a number line for addition, children should be taught to use their understanding of partitioning numbers and number bonds to bridge tens.





ADDITION
Year 2

National Curriculum Objectives: Addition objectives from Addition and Subtraction Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
Solve problems with addition • Using concrete objects and pictorial representations, including those involving numbers, quantities and measures. • Apply their increasing knowledge of mental and written methods. Add numbers using concrete objects, pictorial representations and mentally: • Add two-digit numbers and ones. • Add two-digit numbers and tens. • Add two, two-digit numbers. • Add three one-digit numbers. • Recall and use addition facts up to 20 fluently and derive and use related facts up to 100. • Show that addition can be done in any order. • Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing box number problems.	• Count in steps of 2, 3 and 5 and count in tens from any number • Understand the place value of 2-digit numbers (tens, ones) • Compare and order numbers to 100 and use <, > and = signs. • Read and write numbers to at least 100 in numerals and words. • Identify, represent and estimate numbers using different representations, including the number line.	Addition, add, altogether, put together, sum, and, plus, total Addend Part, whole Equals, equal to, same as More than, less than, fewer, most, least, double Regroup, partition Flexible partitioning

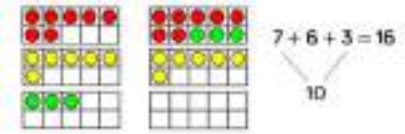
Children should:

- Have access to a wide range of practical resources and equipment, counting materials, models and images and should be given the opportunity to use these to support their understanding of mental and written methods at all stages.

- Spend time learning and practising mental methods for addition, starting with questions which do not cross boundaries and progressing to those that do. This will include exploring number and different ways to partition number to support addition strategies, as well as adding the nearest multiple of 10 and adjusting (e.g. to add 9, 19 etc.), using near doubles and partitioning and recombining.

Mental Methods: The children should be taught to use the following mental strategies, and to use jottings to support their methods:

- Reordering numbers when adding
- Counting on in tens or ones
- Using knowledge of pairs making 10 and place value
- Compensating: add 9, 19, 11 or 21 (by adding 10/20 and adding/subtracting 1)
- Compensating: doubling and adjusting
- Partitioning small numbers to bridge through 10 when adding. e.g. 8+3 = 8+2+1
- Partition and combine multiples of tens and ones
- Use tens frames when adding three single digit numbers before considering which order to add the numbers in. The ten frames support the link to effective mental methods of addition as well as the importance of commutativity



Steps for Written Methods:

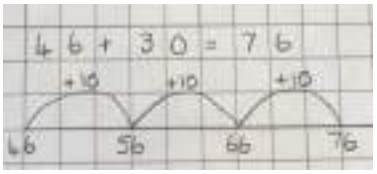
All steps should be supported by concrete and pictorial, with a focus on Year 2 children becoming confident in using a blank number line as a jotting for their mental method as well as a pictorial representation to prepare them for more formal written methods in the future.

Step 1: Add two-digit numbers and ones

Children consolidate and extend their learning from year 1

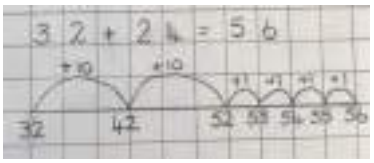
Step 2: Adding tens to a two-digit number

Children build on their previous learning and progress from adding one-digit numbers to adding multiples of 10.



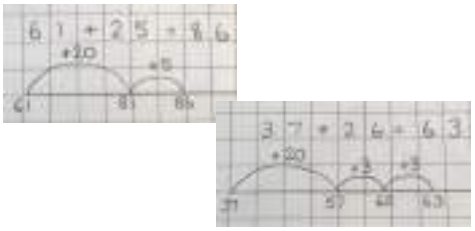
Step 3: Adding two, two-digit numbers

Once confident with the above, and using their ability to partition two-digit numbers into tens and ones, children add two two-digit numbers on a number line.



Step 4: Using number lines efficiently

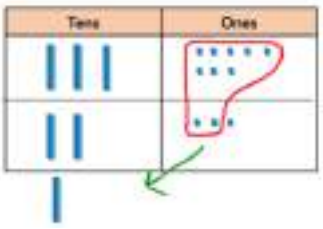
When children are showing a secure understanding of the above steps, they should be taught to use number lines to calculate in the most efficient way. This will include partitioning numbers to bridge tens, as in the second example.

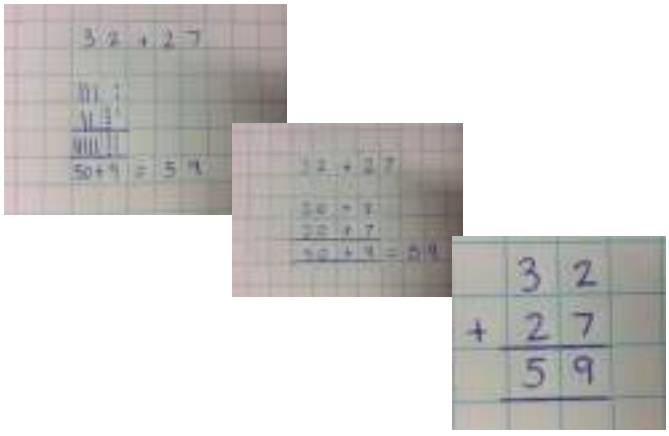


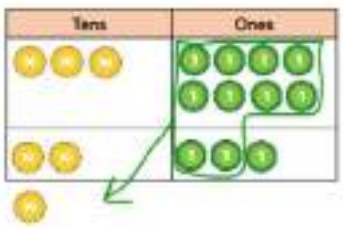
Step 5: Partitioned Column Method (Year 2 **if appropriate** and Year 3)

Those children who are ready, **may** progress to a partitioned column method.

Children should work with examples which do not cross boundaries first before progressing when they are ready to those which do cross boundaries.







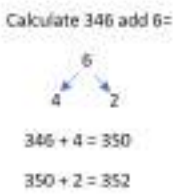
ADDITION
Year 3

National Curriculum Objectives: Addition objectives from Addition and Subtraction Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Add numbers mentally, including three-digit numbers and ones. • Add numbers mentally, including three-digit numbers and tens. • Add numbers mentally, including three-digit numbers and hundreds. • Add numbers with up to 3 digits, using formal written method of columnar addition. • Estimate the answer to a calculation and use inverse operations to check answers. • Solve problems including missing number problems, using number facts, place value, and more complex addition.	• Find 10 or 100 more than a given number. • Recognise the place value of each digit in a three-digit number. • Identify, represent and estimate numbers using different representations. • Read and write numbers up to 1000 in numerals in words. • Compare and order numbers up to 1000.	Hundreds / tens / ones Partition, regroup Addend, sum Mental partitioning Column method Commutative *Plus all KS1 vocabulary: Addition, add, altogether, put together, sum, and, plus, total* *Addend* *Part, whole* *Equals, equal to, same as* *More than, less than, fewer, most, least, double*

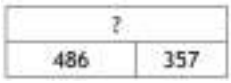
Children should:

- Have access to a wide range of practical resources and equipment, counting materials, models and images and should be given the opportunity to use these to support their understanding of mental and written methods at **all stages**.

- Spend time learning and practising mental methods for addition, starting with questions which do not cross boundaries and progressing to those that do. This will include exploring number and different ways to partition number, to support addition strategies as well as adding the nearest multiple of 10 and adjusting (e.g. to add 9, 99 etc.), using near doubles and partitioning and recombining.



- Use representations, e.g. bar model or part-whole model, to help guide them to the correct operation.



Mental Methods: The children should be taught to use the following mental strategies, and to use jottings to support their methods, where appropriate:

- Counting on in hundreds, tens and ones to find the total
- Partitioning into hundreds, tens, and ones in different ways, then recombining (824 = 800 + 20 + 4, 824 = 700 + 110 +14).
- Reordering numbers when adding.
- Bridging through a multiple of 10, then adjusting
- Using known facts and place value to add
- Using patterns of similar calculations
- Compensating: add or subtract 10, 20 or 100 and adjust.
- Using blank number lines to support partitioning the addend – label number line with appropriate jumps that can be added mentally.

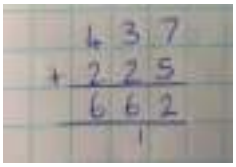
Steps for Written Methods:

Base ten / place value counters should be used in all stages to support children's understanding.

Step 1: To apply expanded column methods from Y2 when adding three-digit numbers. This may be new teaching for the children, depending on whether they were ready for this in Year 2. Therefore, they may need to track back.
Initially regroup in the ones column only.

Step 2: Move to the compact column addition, first without regrouping.

Step 4: Regrouping in the tens column only. Children may need to use the expanded method first.

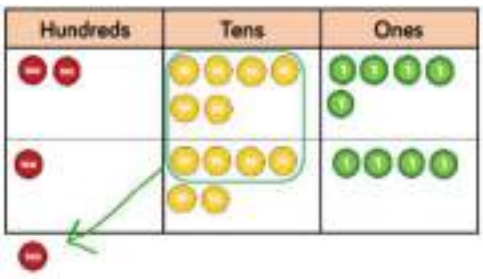


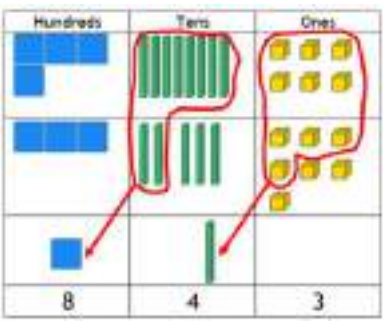
Add the ones first.

Regroup numbers underneath the bottom line.

Step 5: Regrouping in ones and tens columns.

Ensure correct use of vocabulary throughout. E.g. we are adding 3 tens and 8 tens (30 and 80) to make 11 tens or 110, as opposed to 3 and 8 equals 11.





ADDITION
Year 4

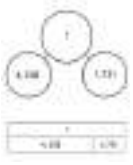
National Curriculum Objectives: Addition objectives from Addition and Subtraction Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Add numbers with up to 4 digits using the formal methods of columnar addition where appropriate. • Estimate and use the inverse operations to check answers to a calculation. • Solve two-step addition problems in contexts, deciding which operations to use and why.	• Find 1,000 more than a given number. • Recognise the place value of each digit in a four-digit number. • Identify, represent and estimate numbers using different representations. • Compare and order numbers beyond 1000.	Thousands / hundreds / tens / ones Partition, regroup Addend, sum Mental partitioning Column method Commutative Inverse, operation, Tenths, hundredths, decimal point, decimals *Plus all previously taught vocabulary.*

Children should:

- Have access to a wide range of practical resources and equipment, counting materials, models and images and should be given the opportunity to use these to support their understanding of mental and written methods at all stages.
- Be given time to extend their previous work on adding multiples of 10 and 100 before adding 1000s as well.
- Spend time learning and practising mental methods for addition. This will include exploring number and different ways to partition number to support addition strategies, as well as adding the nearest multiple of 10, 100 and 1000 and adjusting (e.g. to add 9, 49, 99, 999 etc), using near doubles and partitioning and recombining.

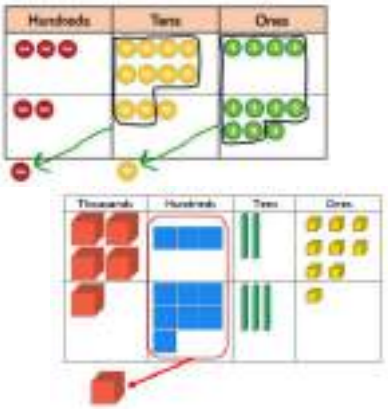
Mental Methods: The children should be taught to use the following mental strategies, and to use jottings to support their methods, where appropriate:

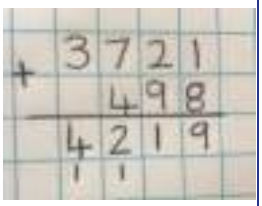
- Counting in steps of thousands, hundreds, tens and ones
- Reordering numbers in a calculation
- Adding 3 or 4 small numbers
- Partitioning: adding the most significant digit first
- Compensating: doubling and adjusting
- Compensating: Adding the nearest multiple of 10 or 100 and then adjust
- Using knowledge of place value and related calculations e.g. working out 150 +140 = 290 by using 15 + 14 = 29.
- Using representations, e.g. bar model or part-whole model, to help guide them to the correct operation.



Written Methods:

Recap the children's understanding of Year 3 calculation strategy, moving to the compact column method adding the ones first and regrouping numbers underneath the calculation.

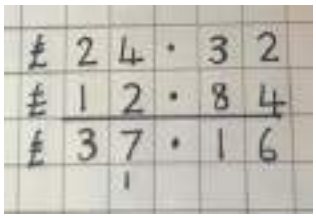


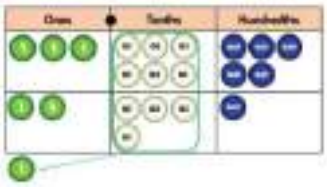


Ensure correct use of vocabulary throughout. E.g. we are adding 7 hundreds and 4 hundreds, not 7 add 4.

Teach children to apply their use of the column methods to decimal numbers.

Apply column method to money and measurement values.





The decimal point should be aligned in the same way as the other place value columns and must be in the same column in the answer.

ADDITION
Year 5

National Curriculum Objectives: Addition objectives from Addition and Subtraction Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Add whole numbers with more than 4 digits, including using formal written methods (columnar addition). • Add increasingly large numbers mentally. • Solve addition multi-step problems in contexts, deciding which operations and methods to use and why.	• Read, write and compare numbers to at least 1,000,000 and determine the value of each digit. • Count forwards in steps of powers of ten for any given number up to 1,000,000.	Ten thousands / thousands / hundreds / tens / ones Partition, regroup Addend, sum Commutative Mental partitioning Column method Inverse, operation, Tenths, hundredths, thousandths, decimal point, decimals *Plus all previously taught vocabulary.*

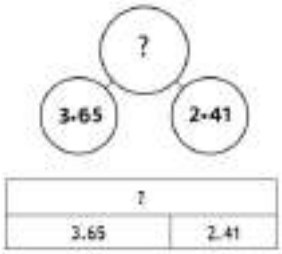
Children should:

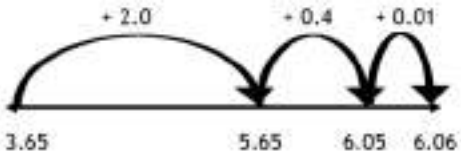
- Have access to a wide range of practical resources and equipment, counting materials, models and images and should be given the opportunity to use these to support their understanding of mental and written methods at all stages.
- Spend time learning and practising mental methods for addition. This will include exploring number and different ways to partition number to support addition strategies, as well as adding the nearest multiple of 10, 100 and 1000 and adjusting (e.g. to add 9, 49, 99, 999 etc.), using near doubles, partitioning and recombining, inverse and using number bonds.

Mental Methods: The children should be taught to use the following mental strategies, and to use jottings to support their methods, where appropriate:

- Counting on in steps of 0.1, 1, 10, 100 or 1,000
- Reordering the numbers in a calculation
- Partitioning, adding the most significant digit first
- Compensating: Add a multiple of 10, 100 or 1,000 and adjust.
- Compensating: Double and adjust.
- Using knowledge of place value and related calculations e.g. 6.3 + 4.8 using 63 + 48

- Using blank number lines to support partitioning the addend – label number line with appropriate jumps that can be added mentally.

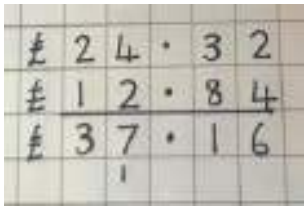




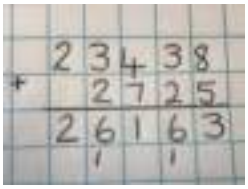
Written Methods:

To include money, measures and decimals with different numbers of decimal places.

The decimal point should be aligned in the same way as their other place value columns and must be in the same column in the answer.

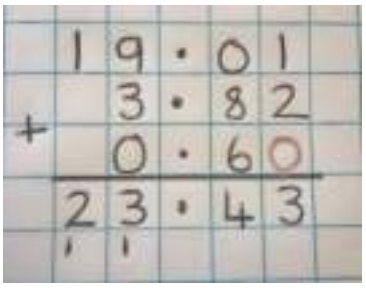


Numbers should exceed 4 digits.



Pupils should be able to add more than two values, carefully aligning place value columns.

Children should understand the place value of tenths and hundredths and use this to align numbers with different numbers of decimal places.



Empty decimal places could be filled with zero to show the place value of the column.

Ensure correct vocabulary is used throughout. E.g. 6 tenths and 7 tenths equals 13 tenths.

ADDITION
Year 6

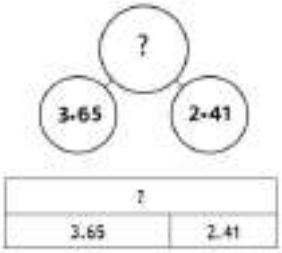
National Curriculum Objectives: Addition objectives from Addition, subtraction, multiplication and division strand.	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Perform mental calculations, including with mixed operations and large numbers. • Use their knowledge of the order of operations to carry out calculations involving the four operations. • Solve addition multi-step problems in contexts, deciding which operations and methods to use and why. • Solve problems involving four-operations. • Use estimation to check answers to calculations.	• Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit.	*Consolidate use of all previously taught vocabulary.*

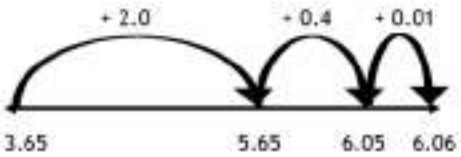
Children should:

- Have access to a wide range of practical resources and equipment, counting materials, models and images and should be given the opportunity to use these to support their understanding of mental and written methods at all stages.
- Spend time learning and practising mental methods for addition. This will include exploring number and different ways to partition number to support addition strategies, as well as adding the nearest multiple of 10, 100 and 1000 and adjusting (e.g. to add 9, 49, 99 999 etc.), using near doubles, partitioning and recombining, inverse and using number bonds.

Mental Methods: The children should be taught to use the following mental strategies, and to use jottings to support their methods, where appropriate.

- Consolidate all strategies from previous years
- Partitioning, adding the most significant digit first
- Compensating: adding a whole number, multiple of 10 or double and adjust.
- Using knowledge of place value and related calculations e.g. 680 + 430, 6.8 + 4.3, 0.68 + 0.43 can all be worked out using the related calculation 68 + 43.
- Using blank number lines to support partitioning the addend – label number line with appropriate jumps that can be added mentally.





Written Methods:

Written method of column addition should be used in a variety of contexts and with numbers of increasing size and complexity.

This should include addition several numbers with different numbers of decimal places and more than four-digits. (including in the context of measures and money)

Tenths, hundredths and thousandths should be correctly aligned, with the decimal place lined up vertically, including in the answer row.

Zeros should be added into empty decimal places to show there is no value to add and to support with place value.

SUBTRACTION

Early Years

Objectives from Early Years Foundation Stage Framework: Number	Objectives from Early Years Foundation Stage Framework: Numerical Patterns	Key Vocabulary
• Have a deep understanding of number to 10, including the composition of each number. • Subitise (recognise quantities without counting) up to 5 • Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.	• Verbally count beyond 20, recognising the pattern of the counting system • Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity • Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	More than, less than, fewer, most, least, same Subtract, take away, how many left Whole, part Equals

Children should:

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts.
- Begin to develop an understanding of subtraction practically, showing subtraction on bead strings, using cubes etc.
- Understand that a group of objects gets smaller if part of the group is taken away from the whole.

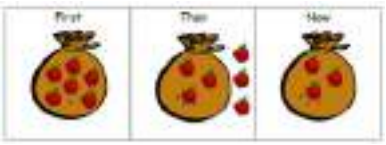
SUBTRACTION

Year 1

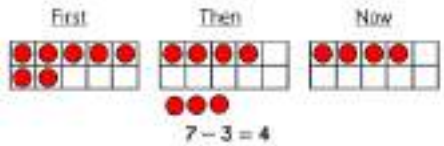
National Curriculum Objectives: Subtraction objectives from Addition and Subtraction Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Read, write and interpret mathematical symbols involving subtraction (-) and equals (=) signs. • Represent and use related subtraction facts within 20. • Subtract one-digit and two-digit numbers to within 20, including 0 • Solve one step problems that involve subtraction, using concrete objects and pictorial representations and missing number problems.	• Count backwards from and across 100, beginning from any given number. • Count, read and write numbers to 100 in numerals. • Given a number, identify one less. • Identify and represent numbers using objects and pictorial representations including the number line. • Read and write numbers from 1-20 in numerals and words.	Subtraction, subtract, take away, minus Halve, half, Difference Less, least, few, fewer, fewest Part, whole Equals Tens, ones

Children should:

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts.
- Consolidate their understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. They should then use practical resources alongside pictures to make the link between concrete and pictorial representations.



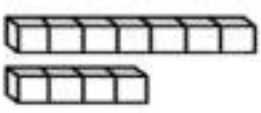
- Use ten frames to look at taking away. This can be introduced with a **first, then, now** structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help the children understand the change e.g. **First**, there were 7 cars. **Then**, 3 cars left. **Now**, there are 4 cars.



- Read and write the subtraction (-) and equals (=) signs within number sentences. Specific learning should take place, through exploration of number and use of practical resources, around the equals sign - ensuring children understand its role within a number sentence and that it does not always just indicate where the answer goes..

- Interpret addition number sentences and solve missing box problems, using concrete objects and number line subtraction to solve them: 8 - 3 = ☐ 15 -☐ = 10 and 9 = 14 - ☐

- Be introduced to finding the difference. This will be introduced practically, using the language 'find the distance between' and 'How many more?'



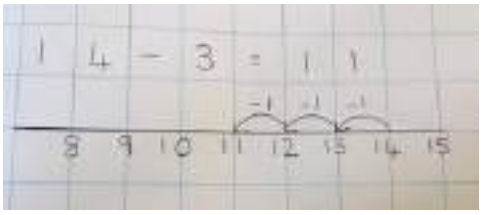
7 is 3 more than 4

Mental Methods: The children should be taught to use the following mental strategies, and to use jottings to support their methods:

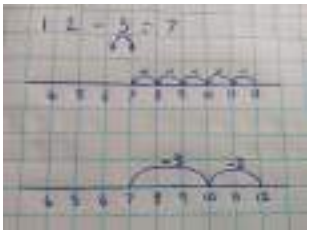
- Counting back in ones
- Finding one less than a number
- Finding 10 less than a multiple of 10
- Taking away a small number by counting back
- Finding a small difference by counting on (using equipment)
- Beginning to bridge through 10 when subtracting a one-digit number
- Starting to recall subtraction facts up to and within 10 and 20 and understand subtracting 0.

Written Methods: Building on the prior learning, and exploration of number outlined above.

Step 1: Use numbered number lines to subtract, by counting back in ones, marking the jumps on a number line or number track



Step 2: Once confident using a number line for subtraction, children should be taught to use their understanding of partitioning numbers and number bonds to bridge tens.



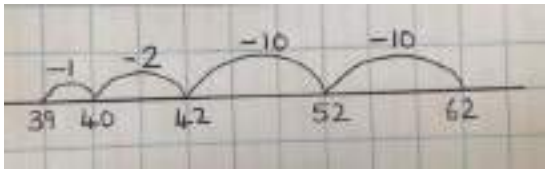
SUBTRACTION

Year 2

National Curriculum Objectives: Subtraction objectives from Addition and Subtraction Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
Solve problems with subtraction • Using concrete objects and pictorial representations, including those involving numbers, quantities and measures. • Apply their increasing knowledge of mental and written methods. Subtract numbers using concrete objects, pictorial representations and mentally: • Subtract two-digit numbers and ones. • Subtract two-digit numbers and tens. • Subtract two, two-digit numbers. • Recall and use subtraction facts within 20 fluently and derive and use related facts up to 100. • Show that subtraction cannot be done in any order. • Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing box number problems.	• Count in steps of 2, 3 and 5 and count in tens from any number • Understand the place value of 2-digit numbers (tens, ones) • Compare and order numbers to 100 and use <, > and = signs. • Read and write numbers to at least 100 in numerals and words. • Identify, represent and estimate numbers using different representations, including the number line.	Subtraction, subtract, take away, minus, decrease Difference Less, least, few, fewer, fewest Part, whole Equals Tens, ones Partition, regroup Subtrahend Inverse

Children should:

- Continue to have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts.
- Consolidate their understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. They should then use practical resources alongside pictures to make the link between concrete and pictorial representations.
- Read and write the subtraction (-) and equals (=) signs within number sentences. Specific learning should take place, through exploration of number and use of practical resources, around the equals sign - ensuring children understand its role within a number sentence and that it does not always just indicate where the answer goes.



- Interpret addition number sentences and solve missing box problems, using concrete objects and number line subtraction to solve them: E.g. 28 - 3 = ☐ 35 - ☐ = 21 and ☐ = 24 – 9
- Be introduced to finding the difference, continuing to develop understanding of this e.g. how many more, how many less etc. Children to start to develop an understanding of the different methods that can be used for this (counting back, counting on) and when we might use each.

Mental Methods: The children should be taught to use the following mental strategies, and to use jottings to support their methods:

- Counting back in tens and ones.
- Subtracting mentally a 'near multiple of 10' by subtracting and adjusting
- Subtracting a small number by counting back.
- Finding a small difference by counting up from the smaller to the larger number (on a number line)
- Recalling subtraction facts for numbers to 20 and using these to derive the related facts up to 100.
- Subtracting by partitioning the second number and subtracting tens then ones.
- Using patterns of similar calculations.
-

Written Methods:

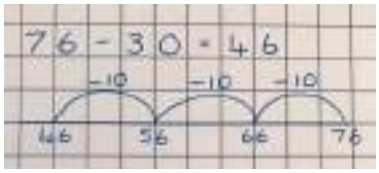
All steps should be supported by concrete and pictorial, with a focus on Year 2 children becoming confident in using a blank number line as a jotting for their mental method as well as using a pictorial representation to prepare them for more formal written methods in the future.

Step 1: Subtracting ones from a two-digit number

Children consolidate and extend their learning from Year 1 and extend to larger 2-digit numbers

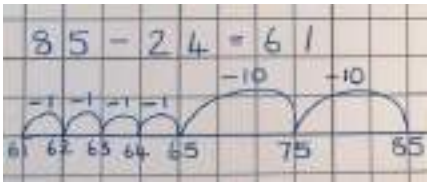
Step 2: Subtracting multiples of ten from two-digit numbers

Children build on their previous learning and progress from subtracting one-digit numbers to subtracting multiples of 10.



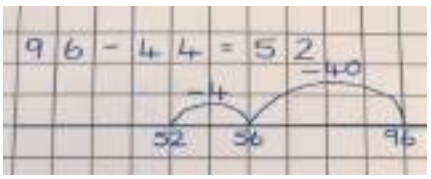
Step 3: Subtracting two, two-digit numbers

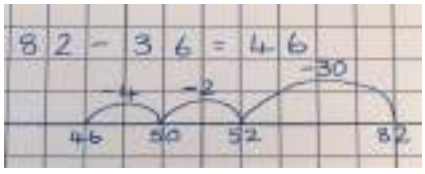
Once confident with the above, and using their ability to partition two-digit numbers into tens and ones, children subtract a two-digit number from a larger two-digit number on a number line.



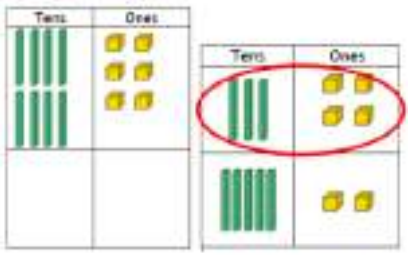
Step 4: More efficient methods for subtraction.

When children are showing a secure understanding of the above steps, they should be taught to use number lines to calculate in the most efficient way. This will include partitioning numbers to bridge tens, as in the second example.





Those children who are ready, may progress to using base ten to support an introduction to the column method. They should work initially with numbers which do not cross boundaries and involve any regrouping.



SUBTRACTION

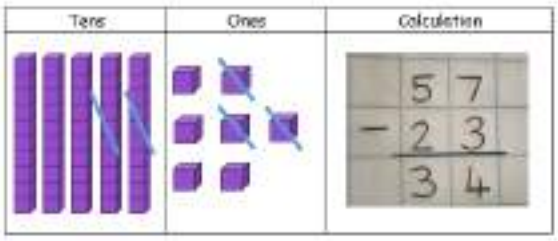
Year 3

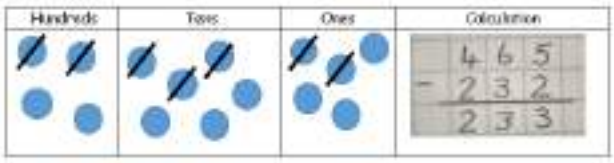
National Curriculum Objectives: Subtraction objectives from Addition and Subtraction Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Subtract numbers mentally, including three-digit numbers and ones. • Subtract numbers mentally, including three-digit numbers and tens. • Subtract numbers mentally, including three-digit numbers and hundreds. • Subtract numbers with up to 3 digits, using formal written method of columnar subtraction. • Estimate the answer to a calculation and use inverse operations to check answers. • Solve problems including missing number problems, using number facts, place value, and more complex subtraction.	• Find 10 or 100 less than a given number. • Recognise the place value of each digit in a three-digit number. • Identify, represent and estimate numbers using different representations. • Read and write numbers up to 1000 in numerals in words. • Compare and order numbers up to 1000.	Difference, subtrahend, minuend Inverse Part, whole, equals Hundreds, tens, ones Partition, regroup *Plus all other KS1 vocabulary: Subtraction, subtract, take away, minus, decrease* *Difference* *Less, least, few, fewer, fewest* *Part, whole, equals*

Children should:

- Continue to use practical resources alongside the developing calculation strategies. These will include base ten, place value counters, place value charts, digit cards etc and will be appropriate for support children's subtraction with three-digit numbers.

- Use concrete resources or pictorial representations alongside their abstract calculations.





- Continue to develop their understanding of finding the difference. They should experience different contexts and representations for doing this and start to become more secure with choosing the most effective strategy.

Mental Methods: The children should be taught to use the following mental strategies, and to use jottings to support their methods:

- Counting back in hundreds, tens and ones.
- Counting on as a mental strategy for subtraction when the numbers are close together (e.g. 131-129), and for finding a small difference.

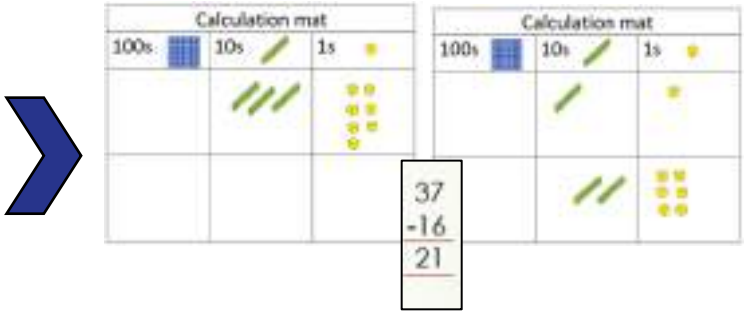
- Compensating: subtract mentally a near multiple of 10 then adjust.
- Bridging through a multiple of 10
- Using knowledge of number facts and place value to subtract pairs of numbers
- Subtracting a two-digit number by partitioning it then subtracting tens and ones.
- Using patterns of similar calculations
- Using the relationship between addition and subtraction
- Using bar models / part-whole models to help interpret subtraction number sentences and solve missing box problems

Written Methods:

Step 1: Subtraction with no regrouping – column method

This may be new teaching for the children, depending on whether they were ready for this in Year 2.

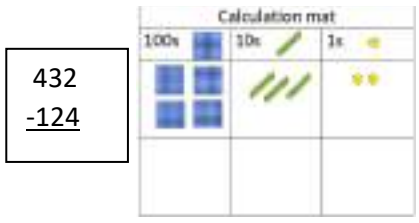
Model how to record this.



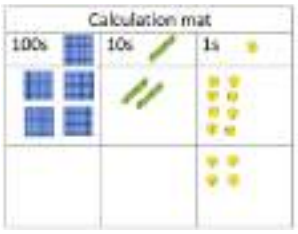
Step 2: Introduce regrouping

Begin with regrouping from the tens only. Make the larger number with base ten and then physically regroup a tens rod for ten ones. Model how to record this.

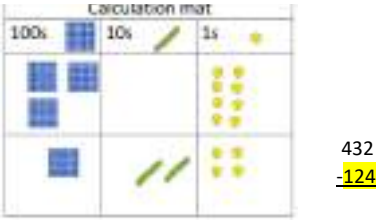
Write the calculation and then make the first number using base ten and the calculation mat.



As you cannot subtract 4 from 2, you need to regroup a ten into ten ones before subtracting 4 ones.



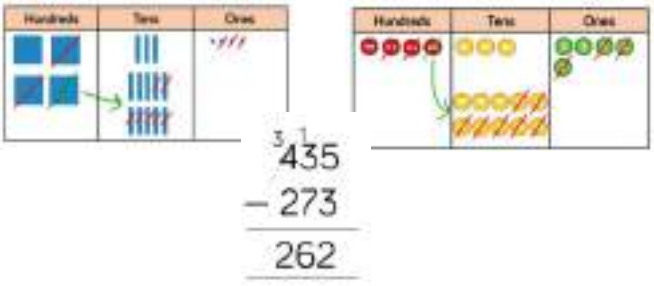
Subtract 2 tens and then 1 hundred.



Check the correct value (124) has been subtracted.

Step 3: Compact column method

Once the children are secure with regrouping, they can use this to subtract two and three-digit numbers in a variety of contexts.



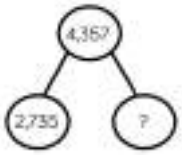
SUBTRACTION

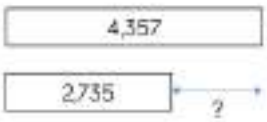
Year 4

National Curriculum Objectives: Subtraction objectives from Addition and Subtraction Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Subtract numbers with up to 4 digits using the formal methods of columnar addition where appropriate. • Estimate and use the inverse operations to check answers to a calculation. • Solve two-step addition problems in contexts, deciding which operations to use and why.	• Find 1,000 more than a given number. • Recognise the place value of each digit in a four-digit number. • Identify, represent and estimate numbers using different representations.	Inverse Thousands, hundreds, tens, ones Partition, regroup Subtrahend, minuend, difference Tenths, hundredths, decimal point *Plus all other KS1 vocabulary: Subtraction, subtract, take away, minus, decrease* *Difference* *Less, least, few, fewer, fewest* *Part, whole, equals*

Children should:

- Continue to use concrete and pictorial resources to build their understanding as they apply their previous learning to subtracting four-digit numbers.
- Be given the opportunity to explore and use different written and mental strategies in a range of contexts, discussing how effective the strategies are and why. Children should start to select the most effective strategies.
- Continue to develop their understanding of finding the difference, choosing the most appropriate strategies to do so.
- Use bar models / part-whole models to help interpret subtraction number sentences and solve missing box problems.





- Recognise where using a number line is a more effective method e.g. when finding change from £10 or £20.

Mental Methods: The children should be taught to use the following mental strategies, and to use jottings to support their methods:

- Counting on and back in thousands, hundreds, tens, ones.
- Using known facts and place value to subtract
- Counting on to subtract when the numbers are close together.
- Finding a difference by counting up through the next multiple of 10, 100 and 1,000
- Compensating: Subtracting the nearest multiple of 1, 10, 100 or 1,000 and adjusting.
-

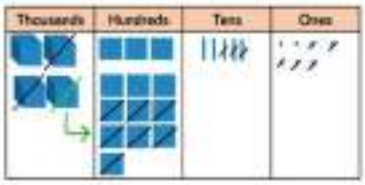
Written Methods: Compact Column Subtraction

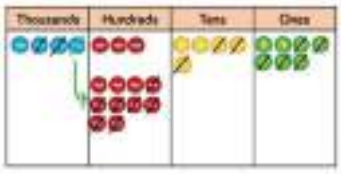
Step 1: Subtraction with no regrouping

Reinforce previous learning from year 3 and extend to subtracting four-digit numbers with no regrouping.

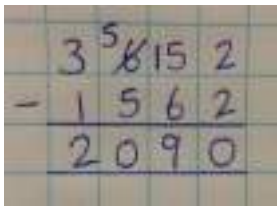
Step 2: Subtraction with regrouping

Move onto regrouping into the ones column only, the tens column only and then the hundreds column only. Extend to regrouping into combinations of the ones, tens and hundreds columns.



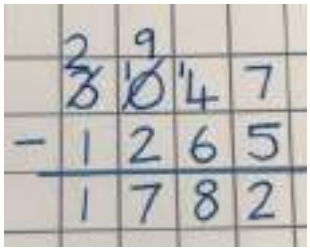


$$\begin{array}{r} {}^{3}\cancel{4}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$



Ensure children have the opportunity to apply this method to a variety of different contexts, including money and measures.

Ensure children have experience of using this method for subtraction where there is a 0 in the column they need to regroup from, and that they understand, through clear modelling (using practical resources) how to move to the next column and regroup then 'move' the value along.



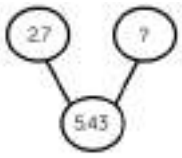
SUBTRACTION

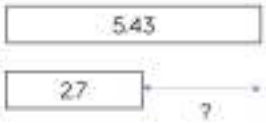
Year 5

National Curriculum Objectives: Subtraction objectives from Addition and Subtraction Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction). • Subtract increasingly large numbers mentally. • Solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why.	• Read, write and compare numbers to at least 1,000,000 and determine the value of each digit. • Count backwards in steps of powers of ten for any given number up to 1,000,000.	Subtrahend, minuend, difference Inverse Partition, regroup Thousands, hundreds, tens, ones Tenths, hundredths, thousandths, decimal point *Plus all previously taught vocabulary.*

Children should:

- Continue to be given the opportunity to work with practical resources and pictorial representations in order to consolidate and extend their understanding of the maths which underpins the methods. They should be interpreting pictorial representations as part of their varied fluency, reasoning and problem solving.
- Become, through regular practise, discussion and modelling, more independent in their ability to select the most efficient methods. They should be given regular opportunities to solve the same problem in several different ways and discuss their findings (this includes finding the difference).
- Use bar models / part-whole models to help interpret subtraction number sentences and solve missing box problems.





- Recognise where using a number line is a more effective method e.g. when finding change from £10 or £20.

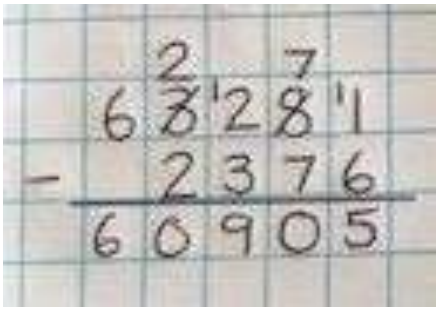
Mental Methods: The children should be taught to use the following mental strategies, and to use jottings to support their methods:

- Counting back steps of 0.1, 1, 10, 100 or 1000
- Using known facts and place value to subtract
- Finding a difference by counting on through the next multiple of 10, 100 or 1,000
- Subtracting by counting up from the smaller to the larger number where this is the most efficient method
- Subtracting the nearest multiple of 1, 10 or 100 then adjust
- Using knowledge of place value and related calculations e.g. 4.5 - 3.6 using 45 - 36
- Using the relationship between addition and subtraction

Written Methods: Compact Column Subtraction

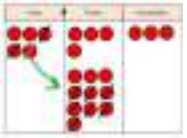
Children use this method to subtract increasingly large and complex numbers, in a range of contexts.

Those children who are not secure should use practical resources to support understanding,

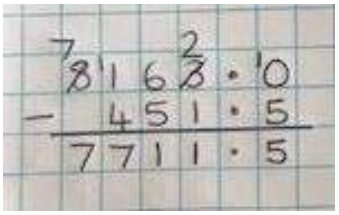


Children should be taught to use this method to subtract decimals, including mixtures of whole numbers and decimals, ensuring they align the decimal point correctly. Use place value counters to support children's understanding.

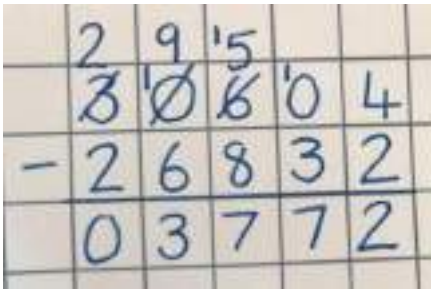
5.43 – 2.7 = 2.73



Children should be taught to add a zero in any empty decimal places to aid understanding of what to subtract in that column.



Ensure children have experience of using this method for subtraction where there is a 0 in the column they need to exchange from, and that they understand, through clear modelling (using practical resources) how to move to the next column and exchange then 'move' the value along.



Extend to provide children with opportunities for subtracting and finding the differences with money and measures.

SUBTRACTION

Year 6

National Curriculum Objectives: Subtraction objectives from Addition, Subtraction, Multiplication and Division Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Perform mental calculations, including with mixed operations and large numbers. • Use their knowledge of the order of operations to carry out calculations involving the four operations. • Solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why. • Solve problems involving four-operations. • Use estimation to check answers to calculations.	• Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit.	*Consolidate use of all previously taught vocabulary.*

Children should:

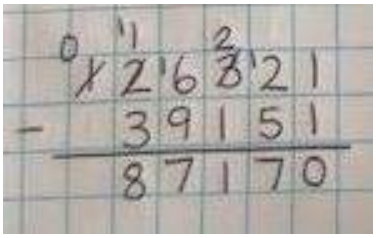
- Be given the opportunity to apply their previous learning into new contexts and use it to solve problems in different ways.
- Where children are not confident with previous learning or do not have a solid understanding of the place value and maths underpinning the strategies, they should have the opportunity to track back and fill these gaps in their learning.
- Children should be given the opportunity to regularly and independently select the most effective method from their repertoire.
- Children should be given the opportunity to interpret and use mathematical pictures and practical resources as part of their varied fluency, reasoning and problem solving.

Mental Methods: The children should be taught to use the following mental strategies, and to use jottings to support their methods:

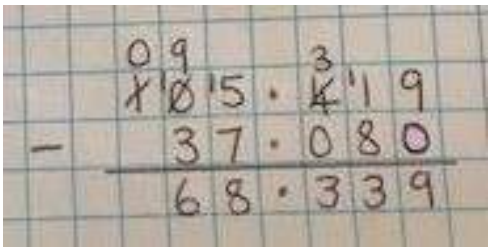
- Consolidating all mental strategies from previous year groups.
- Counting back in powers of tens, including tenths, hundredths and thousandths.
- Using knowledge of place value and related calculations
- Subtracting a power of ten, or a whole number and adjust.
- Finding the difference by counting up through the nearest multiple of 0,1, 10, 100 or 1,000 then adjust.
- Continuing to use the relationship between addition and subtraction.

Written Methods:

Use the compact column method to subtract more complex numbers.



Use the compact column method to subtract money and measures, including decimals with different numbers of decimal places. Children can fill empty decimal places with zeros to show the place value in each column.



Ensure children have experience of using this method for subtraction where there is a 0 in the column they need to exchange from, and that they understand, through clear modelling (using practical resources) how to move to the next column and exchange then 'move' the value along.

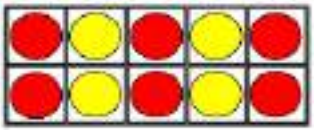
MULTIPLICATION

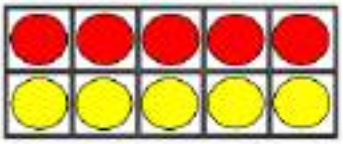
Early Years

Objectives from Early Years Foundation Stage Framework: Number	Objectives from Early Years Foundation Stage Framework: Numerical Patterns	Key Vocabulary
• Have a deep understanding of number to 10, including the composition of each number; • Subitise (recognise quantities without counting) up to 5. • Automatically recall some number bonds to 10, including double facts.	• Verbally count beyond 20, recognising the pattern of the counting system • Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity • Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	Pair Group Double Equal Same as Twos, fives, tens

Children should:

- Have real, practical experiences of handling groups of objects.
- Experience counting equal groups of objects in 2s, 5s and 10s, such as socks (pairs) or fingers (5s or 10s).
- Be given the opportunity to explore and understand the vocabulary of early multiplication e.g. lots of, how many altogether etc.





MULTIPLICATION

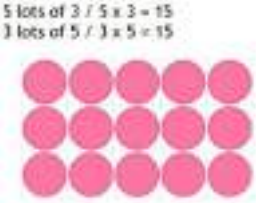
Year 1

National Curriculum Objectives: Multiplication objectives from Multiplication and Division Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays, with support from the teacher.	• Count in multiples of 2,5 and 10.	Pair Groups of, lots of, sets of Double, half Equal, same as Twos, fives, tens Times, altogether, multiply, count Array Skip counting

Children should:

- Begin to understand multiplication by multiplying with concrete objects, arrays and pictorial representations.
- Experience counting equal groups of objects in 2s, 5s and 10s.
- Experience practical problem-solving activities in various contexts.
- Make connections between concrete resources and pictorial representations, number patterns, arrays and counting in 2, 5 and 10s.
- Be given the opportunity to explore and understand the vocabulary of early multiplication e.g. lots of, how many altogether etc.

Mental Methods:



- Counting in multiples of 2, 5, and 10s.
- Spotting number patterns when counting in 2, 5 and 10s.
- Repeated addition
- Links to doubling
- Use of arrays

How many legs will 5 children have?

2 + 2 + 2 + 2 + 2

There are _ groups of _ flowers.

There are _ flowers altogether.

MULTIPLICATION

Year 2

National Curriculum Objectives: Multiplication objectives from Multiplication and Division Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Recall and use multiplication facts for the 2, 5 and 10 times tables. • Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication signs and equals signs. • Show that the multiplication of two numbers can be done in any order. • Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication facts, including problems in context.	• Recognise odd and even numbers. • Count in steps of 2,3 and 5 from zero and in 10s from any number.	Pair Groups of, lots of, sets of Double, half Equal, same as Twos, fives, tens Times, altogether, multiply, count Array, multiplied by, repeated addition, Two times, three times, five times, ten times Skip counting Factor, product

Children should:

- Develop their understanding of multiplication through the use of practical resources and pictorial representations.
- Multiply using arrays and repeated addition.

Mental Methods:

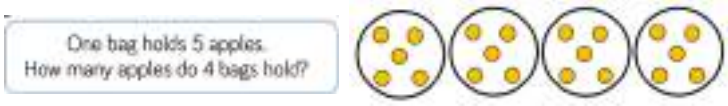
- Counting in twos, fives and tens
- Repeated addition
- Use of arrays
- Children should recall multiplication facts for the 2, 5 and 10 times tables through practising counting and understanding of the operation and number patterns.
- Using doubling and understanding that this is the same as multiplying by 2.
- Reordering a calculation, knowing that multiplication can be done in any order.

Written Methods:

Step 1: Practical Apparatus

Children continue to explore multiplication through use of real-life problems using a range of practical equipment.

5 x 3 = 5 + 5 + 5



Step 2: Arrays

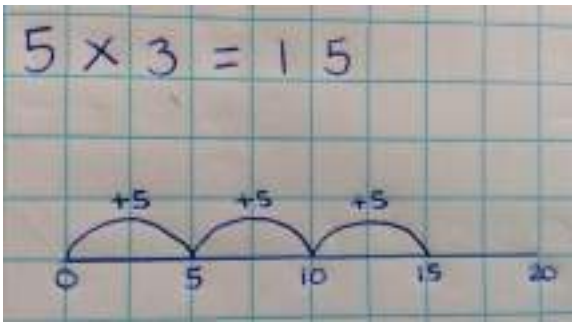
Use arrays to help teach the children that multiplication can be done in any order and explore examples such as 2 x … = 10

2 x 5 = 2 + 2 + 2 + 2 + 2

5 x 2 = 5 + 5

Step 3: Repeated Addition on a number line.

Children start from 0 and make equal jumps on a number line in order to work our multiplication facts and write multiplication statements using x and = symbols.



MULTIPLICATION

Year 3

National Curriculum Objectives: Multiplication objectives from Multiplication and Division Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Recall and use multiplication and division facts for the 3,4 and 8 times tables. • Write and calculate multiplication using the multiplication tables they know, including for two-digit numbers times one-digit numbers, suing mental methods and progressing to formal written methods. • Solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and corresponding problems in which n objects are connected to m objects.	• Count from 0 in multiples of 4 and 8.	Product, multiple Factor Commutative Pair Groups of, lots of, sets of Double, half Equal, same as Twos, fives, tens Times, altogether, multiply, count Array, multiplied by, repeated addition, Two times, three times, five times, ten times

Children should:

- Continue to develop their understanding of multiplication through use of practical resources and pictorial representations. Year 2 multiplication must be consolidated for those children who do not have a secure understanding.
- Be given the opportunity to practise their recall of the 2,5 and 10 times tables.
- Start to be introduced to the grid method for multiplication. However, in order to do this, children need secure understanding of the maths which will underpin this.

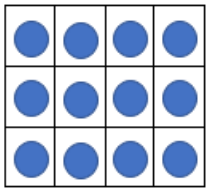
Mental Methods:

- Counting in 2s, 5s, 10s, 3s, 4s and 8s.
- Repeated addition
- Recall multiplication facts for 2, 5 and 10 times tables (from Year 2)
- Recall multiplication facts for 3, 4 and 8 times tables
- Use known facts and place value to multiply by 2,3,4,5,8 and 10.
- Use doubles to link to x2, x4 and x8.
- Reorder a calculation, understanding that multiplication can be done in any order.

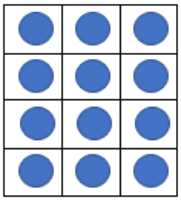
Written Methods for multiplying a two-digit number by a one-digit number.

Step 1: Arrays

Reinforce Year 2 work on arrays, ensuring children have a secure understanding and can apply these to calculate facts for the 3, 4 and 8 times tables.



4 x 3 = 12

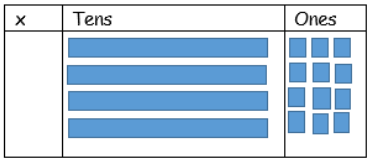


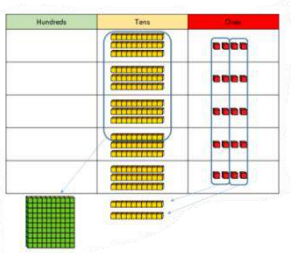
3 + 3 + 3 + 3 = 12

3 x 4 = 12

Step 2: Introducing the grid method

Introduce the grid method to the children by using base ten or place value counters to represent the multiplication statement. E.g. "We need 4 rows of 10 and 4 rows of 3".



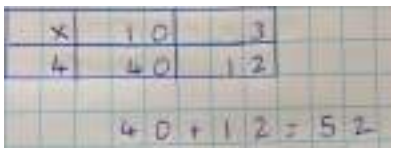


Children can then represent the work they have done with the practical resources, in a way in which they understand, after modelling by the teacher.

Step 3: Grid method

Once the children have a secure understanding of the above steps, the grid method can be introduced, alongside a pictorial representation to start with, then the children practise and use this in a variety of different contexts.

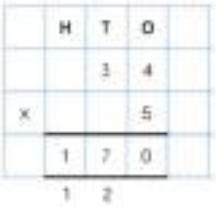
X	10	4
3	30	12



Step 4: Short multiplication

For those children who show a secure understanding of the previous steps and can use these in a variety of contexts, they may be shown how to record this as a short multiplication method. This should be done alongside the grid method so that children are clear on the link between the two.

Expanded method



Compact method

When moving from the grid method to short multiplication, ask children to identify what is the same and what is different about the two methods so they can see the links between them.

MULTIPLICATION

Year 4

National Curriculum Objectives: Multiplication objectives from Multiplication and Division Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Recall multiplication facts for multiplication tables up to 12 x 12. • Use place value, known and derived number facts to multiply mentally including multiplying by 0 and 1. • Multiply 3 numbers together • Multiply two-digit and three-digit numbers by a one-digit number using formal written layout. • Solve problems involving multiplication	• Count in multiples of 6, 7, and 9	Product, multiple, factor, factor pair Commutative Inverse Partition, recombine *Plus previous KS1 vocabulary: Groups of, lots of, sets of* *Double, half, pair* *Equal, same as* *Times, altogether, multiply, count, array, multiplied by, repeated addition,*

Children should:

- Be taught specifically, through exploration of place value, to multiply by 10, 100 and 1,000. Children should be confident in discussing the place value of each digit and how these change. Use a place value grid to support children's understanding.
- Have the opportunity to apply their known number facts to solve other calculations. E.g. if 7 + 4 = 11, then 70 + 40 =, 700 + 400 = etc.
- Be given the opportunity to consolidate and practise their previous learning on multiplication before new content is introduced.
- Be practising their recall of their previously learnt times tables (2, 5, 10, 3, 4 and 8) from the start of year 4, before any new times tables are introduced.
- Be given regular opportunity to approximate before they calculate and use this to check the accuracy of their calculations.

Mental Methods:

- Counting in 6s, 7s, 9s, 25s and 100s
- Recall previously learnt multiplication facts with increasing confidence (2, 5, 10, 3, 4 and 8 times tables).
- Recall multiplication facts for the 6, 7, 9, 11 and 12 times tables – they should then be able to recall all multiplication facts to 12 x 12.
- Partitioning: multiplying hundreds, tens and ones separately and then recombining.
- Using understanding of when a number is multiplied by 10, 100 or 1,000.
- Using knowledge of number facts and place value e.g. 7 x 8 = 56 to find 70 x 8, 7 x 80 etc.

Written Methods:

For progression in multiplying a two-digit number by a one-digit number, see year3.

It may be necessary to revise using arrays in the grid method as a first step to multiplying a 3-digit number by a one-digit number- See Year 3

Step 1: Grid method for multiplying three-digit numbers by a one-digit number.

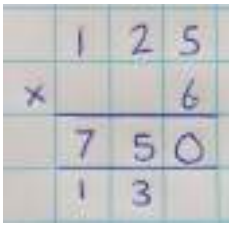
Recap previous multiplication using the grid method and extend this to multiplying two-digit and three-digit numbers by a one-digit number. Track back to using arrays for any children who are not confident.

X	100	30	6
4	400	120	24

400 + 120 + 24 = 544

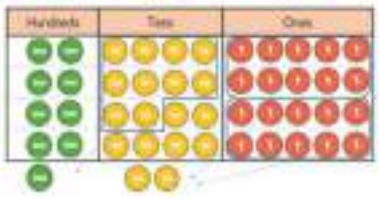
Step 2: Short multiplication for multiplying by a one-digit number.

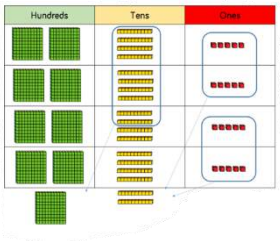
Pupils can be asked to work out a calculation using the grid method, and then compare to the column method..

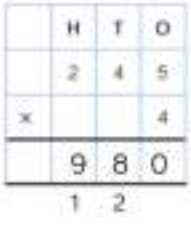


Children should be able to see and understand the links between the different methods.
What are the similarities and differences?
Unpick the steps together and discuss which is the most efficient.

Practical resources can still be used to support children with their calculations.







MULTIPLICATION

Year 5

National Curriculum Objectives: Multiplication objectives from Multiplication and Division Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Multiply numbers up to 4 digits by a one-digit or two-digit number including long multiplication for multiplying by two-digit numbers. • Identify multiples and factors • Multiply mentally, drawing upon known facts. • Multiply whole numbers and those involving decimals by 10, 100 and 1,000. • Recognise and use square and cube numbers. • Solve problems using the 4 operations, and a combination of these, including understanding the meaning of the equals sign. • Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.	• Count forwards in steps of powers of 10 for any given number up to 1,000,000.	Square number, cube number integer Short multiplication, long multiplication Product, multiple, factor Commutative Inverse Partition, recombine *Plus previous KS1 vocabulary*

Children should:

- Recall the multiplication facts for ALL their times tables up to 12 X 12. Children should be given regular opportunities to increase their speed and confidence with this, as well as apply these facts to other calculations.
- Be taught specifically, through exploration of place value, to multiply by 10, 100 and 1,000. Children should be confident in discussing the place value of each digit and how these change.
- Have the opportunity to apply their known number facts to solve other calculations. E.g. if 7 + 4 = 11, then 70 + 40 =, 700 + 400 = etc.
- Be given regular opportunity to approximate before they calculate and use this to check the accuracy of their calculations.

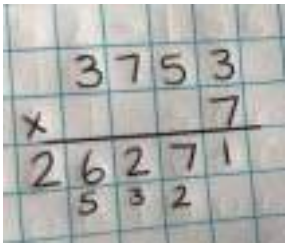
Mental Methods:

- Counting in steps of powers of 10
- Using commutativity and tables to multiply
- Using known facts and place value to multiply
- Using related facts to multiply
- Scaling up using known facts to multiply
- Recalling all times tables up to 12 X 12
- Using times table facts to recognise and use square and cube numbers.
- Using understanding of multiplying by 10, 100 or 1,00 and how the digits change in their place value.
- Using the relationship between multiplication and division.

Written Methods:

Step 1: Short multiplication for multiplying by a one-digit number

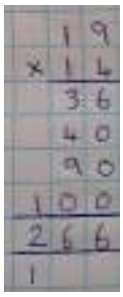
Children use this method to multiply four-digit numbers by a one-digit number, in a range of contexts and units. You may need to return to grid method or use concrete and pictorial for those children not yet secure.

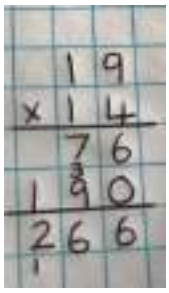


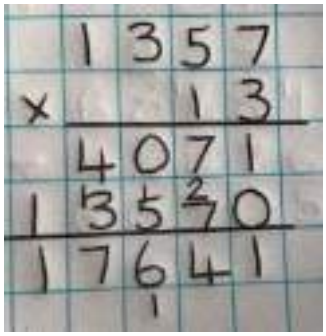
Step 2: Introduce long multiplication for multiplying up to four-digit numbers by two-digit numbers The grid method can be used to introduce long multiplication as this method not only shows each row clearly but will be a familiar method to the children.

Children can be taught to put the '0' in the ones column when multiplying by the tens number, then think '1 times 8, 1 times 1' etc., **as long as they understand the place value involved.**

x	10	9
10	100	90
4	40	36

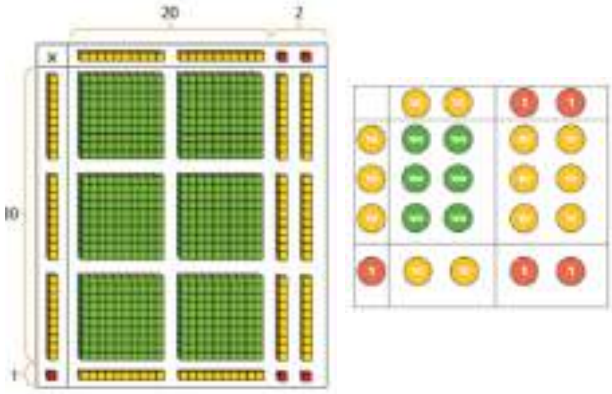






When multiplying a multi-digit number by 2-digits, using the area model shown with base 10 can help children understand the size of the numbers they are using.

31 x 22 = 682



234 x 32 = 7488

MULTIPLICATION

Year 6

National Curriculum Objectives: Multiplication objectives from Addition, subtraction, multiplication and division strand.	Key Skills/ other linked NC Objectives (Place Value / Fractiona)	Key Vocabulary
• Multiply numbers up to 4-digits by 2-digit numbers using long multiplication. • Perform mental calculations, including with mixed operations and large numbers. • Identify common factors and common multiples. • Use their knowledge of the other of operations to carry out calculations involving the four operations. • Solve problems involving the four operations. • Use estimation to check answers to calculations.	• Understanding place value in large numbers • Multiply one-digit numbers with up to two decimal places by whole numbers	Tenths, hundredths, decimals Square number, cube number integer Product, multiple, factor Commutative Inverse Indices *Plus all previous vocabulary*

Children should:

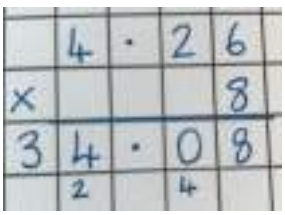
- Have the opportunity to consolidate previous multiplication work and track back if they are not secure.
- Have the opportunity to apply short and long division to various contexts and use it as part of their varied fluency, reasoning and problem solving.
- Be given regular opportunity to approximate before they calculate and use this to check the accuracy of their calculations.

Mental Methods:

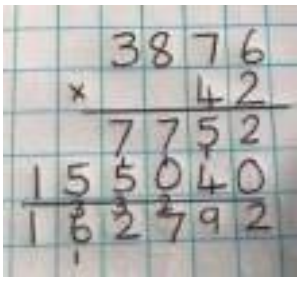
- Rapid recall of all times tables up to 12x12 - as in Year 4 and Year 5
- Recalling square and cubed numbers
- Using known facts and place value to multiply.
- Using related facts to multiply.
- Scaling up using known facts.
- Using the relationship between multiplication and division.

Written Methods: Short and long multiplication

Children will use short multiplication to multiply numbers with more than 4 digits by a 1-digit number, to multiply money and measures and to multiply decimals with up to 2 decimal places by a single digit.



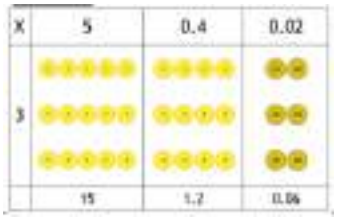
Children will use long multiplication to multiply numbers with up to 4-digits by 2-digit numbers.



Multiplying decimals

5.42 x 3 = 15 + 1.2 + 0.06 = 16.26

Children will use place value counters and the grid method to develop an understanding of multiplying decimals.



X	5	0.4	0.02
3	15	1.2	0.06

5.42 x 3 = 15+1.2+0.06 = 16.26

Once secure, the children can then move onto the column method.

$$\begin{array}{r} 5.42 \\ \times \quad 3 \\ \hline 16.26 \\ {\scriptstyle 1\ 1} \end{array}$$

DIVISION

Early Years

Objectives from Early Years Foundation Stage Framework: Number	Objectives from Early Years Foundation Stage Framework: Numerical Patterns	Key Vocabulary
• Have a deep understanding of number to 10, including the composition of each number; • Subitise (recognise quantities without counting) up to 5. • Automatically recall some number bonds to 10, including double facts.	• Verbally count beyond 20, recognising the pattern of the counting system • Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity • Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	Group Equal, not equal Share Same as One each

Children should:

- Have real, practical experiences of handling groups of objects.
- Experience counting groups of objects and sharing them equally

DIVISION

Year 1

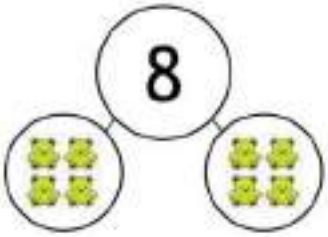
National Curriculum Objectives: Division objectives from Multiplication and Division Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays, with support from the teacher.	• Counting in 2s, 5s and 10s	Group Equal, not equal Share, share equally Same as One each, two each Groups of, lots of Array

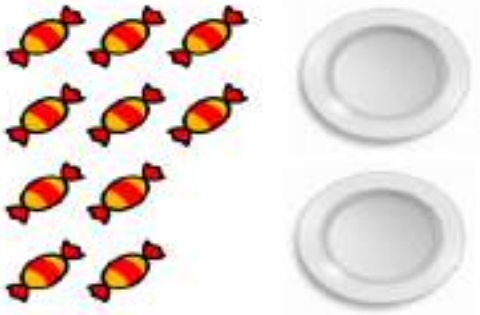
Children should:

- Be given lots of opportunity to explore division as both grouping and sharing, using practical resources and pictorial representations to solve simple problems.
- Be taught to understanding the difference between grouping objects (How many groups of 2 can you make?) and sharing objects (Share these sweets between two people).
- Be taught to find half of a group of objects by sharing into 2 equal groups.
- Children should be taught to interpret and use pictures to support their grouping and sharing, alongside the use of practical objects and resources.

Mental Methods:

- Counting in twos, fives and tens
- Making links to halving
- Using arrays
- Through grouping and sharing small quantities, children will begin to understand division and finding simple fractions of objects, numbers and quantities:





Share the sweets equally between the two plates.

How many equal groups of 2 can you make with the pencils?

If you had 12 pencils, how many groups of 2 would be able to make?

DIVISION

Year 2

National Curriculum Objectives: Division objectives from Multiplication and Division Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Recall and use division facts for the 2, 5 and 10 times tables. • Calculate mathematical statements for division within the multiplication tables and write them using the multiplication signs and equals signs. • Show that the division of two numbers cannot be done in any order. • Solve problems involving division, using materials, arrays, repeated addition, mental methods and multiplication facts, including problems in context.	• Counting in 2s, 5s, 10s and 3s.	Group, groups of, lots of Equal, not equal Share, share equally Array Division, divided by, shared by, Grouping, sharing, left, left over Repeated subtraction

Children should:

- Have plenty of opportunities to use objects, arrays and pictorial representations to group and share.
- Develop their understanding of the divide and equals signs through recording their practical activities and exploration.
- Develop an understanding of **sharing** and **grouping** and understand what is the same and what is different about these.

Mental Methods:

- Counting in 2s, 5s, 10s and 3s
- Making links to arrays
- Recalling the division facts for the 2 ,5 and 10 times tables
- Using knowledge that halving is in the inverse of doubling and the same as dividing by 2.
- Using known facts and place value to divide.

Written Methods:

Step 1: Understanding Arrays and the link between multiplication and division Children should be taught to interpret arrays and use these to understand the link between multiplication and division. For example, by being able to generate the 4 linked multiplication and division sentences.

10 ÷ 2 = 5 2 x 5 = 10

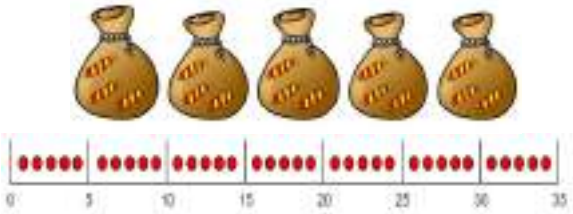
10 ÷ 5 = 2 5 x 2 = 10

Step 2: Practical problem solving with a focus on recognising grouping and sharing.
Children should be given the opportunity to recap their practical work on sharing and grouping, with a focus on understanding the difference and being taught to recognise whether problems involve grouping or sharing.

Sam has 20 sweets, which she shares equally between 5 friends. How many will each friend get?

Children can use pictures or shapes to divide quantities and start to record the division number sentence alongside these.

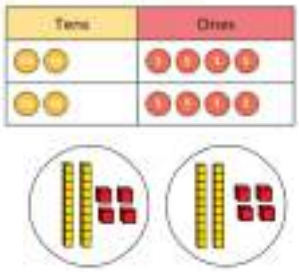
Children to experience grouping in various different contexts, to ensure they are confident with the concept before looking at grouping using a number line.

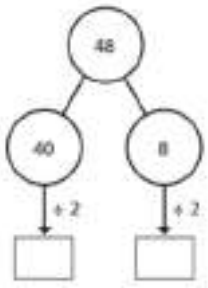


Step 3: Using a number line Children use a number line, by jumping in equal amounts to find out for example, how many groups of 5 there are in 20, as a response to questions such as: I have 20 sweets, I share them between 5 children. How many sweets do they have each?

48 ÷ 2 =

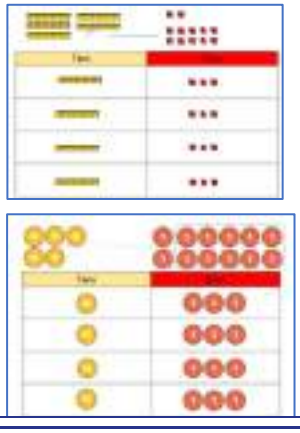
Step 4: Divide two-digit numbers by a one-digit number (no regrouping)
Children should use manipulatives that can be partitioned into tens and ones.

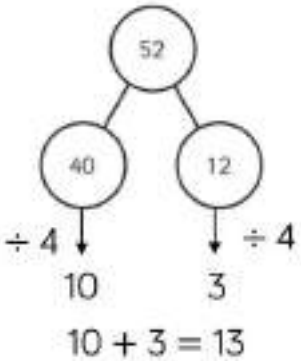




52 ÷ 4 = 12

Step 5: Divide two-digit numbers by a one-digit number (with regrouping)
Children should use manipulatives to regroup tens into ones. Begin with calculations where only one ten will need regrouping.





Flexible partitioning using a part-whole model can support children.

DIVISION

Year 3

National Curriculum Objectives: Division objectives from Multiplication and Division Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Recall the division facts for the 3, 4 and 8 times tables. • Write and calculate division statements using the multiplication tables they know. • Solve problems, including missing number problems, involving division.	• Count in multiples of 4, 8, 50 and 100.	Dividend, divisor, quotient Group, groups of, lots of Equal, not equal Share, share equally Array Division, divided by, shared by, Grouping, sharing, left, left over

Children should:

- Be given the opportunity to explore division in a range of real-life contexts to enable them to start to build a secure understanding of division and recognise when they should use this operation.

Mental Methods:

- Counting in 2s, 5s, 10s, 3s, 4s and 8s
- Recalling the division facts for the 2, 5 and 10 times tables- from Year 2
- Recalling the division facts for the 3, 4 and 8 times tables.
- Using known facts and place value to divide by 2, 3, 4, 5, 8 or 10
- Using knowledge that halving is in the inverse of doubling and the same as dividing by 2. Use this to link to ÷ 2, ÷ 4 and ÷ 8.
- Using known facts/partition in different ways to become more efficient in mental calculations: e.g. 39 ÷ 3 by taking 3 lots of 10 away mentally, then 3 lots of 3 to get 13 as the answer.
- Use the relationship between multiplication and division.

Written Methods:

Step 1: Divide two-digit numbers by a one-digit number (with and without regrouping)

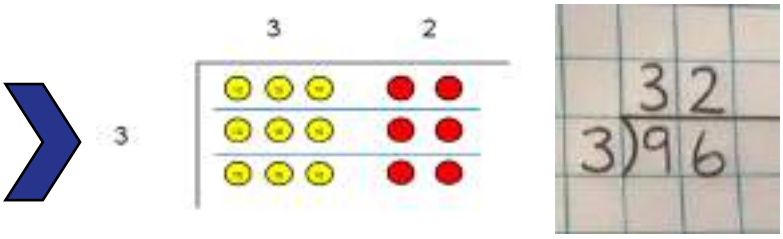
Reinforce Year 2 work on division, ensuring children have a secure understanding of grouping.

Children explore, through the continued use of practical equipment, pictures and number lines, the concept of remainders, how many are left etc. This is preparation for carry remainders across within short division.

Children also continue to develop their understanding of using grouping on a number line to divide, and also to find remainders.

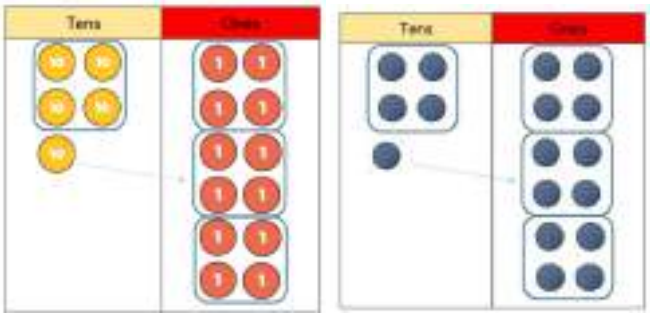
Step 2: Introducing short division (no remainders and no numbers regrouped)

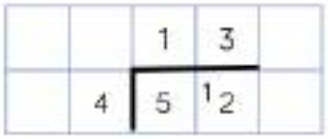
Once children are secure with division as grouping and sharing, using number lines, arrays etc. short division for larger two-digit numbers can be introduced. To start with, this should be introduced with numbers that have no remainders within, or at the end of the calculation.



Step 3: Short division, with no remainders in the final answer

Once children have shown a secure understanding of the above 2 steps, they should be taught how to use short division when remainders occur within the calculation and be shown how to carry the remainder onto the next digit.





Language is important here:

Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

Remainders can also be seen as they are left ungrouped.

DIVISION

Year 4

National Curriculum Objectives: Division objectives from Multiplication and Division Strand	Key Skills/ other linked NC Objectives (Place Value)	Key Vocabulary
• Recall division facts for multiplication tables up to 12x12 • Use place value, known and derived facts to divide mentally- including dividing by 1. • Recognise and use factor pairs and commutativity in mental calculations.	• Counting in 6s, 7s, 9s, 25s and 1000s	Dividend, divisor, quotient Divisible by, factor Group, groups of, lots of Equal, not equal Share, share equally Array Division, divided by, shared by, Grouping, sharing, left, left over Remainder Short division

Children should:

- Continue to develop their understanding of division and extend their previous learning to dividing 3-digit numbers by a single digit.
- Be given a wide variety of real-life contexts to work with and problem solve to help develop a secure understanding of division and how to apply it to problems.
- Understand that a remainder cannot be larger than the divisor

Mental Methods:

- Counting in 6s, 7s, 9s, 25s and 1000s.
- Recalling division facts for all the times tables, up to 12x12
- Using understanding of place value and what happens to the value of each digit when it is divided by 10, 100 or 1,000.
- Using known facts and place value to solve calculations and to become more efficient in mental calculations e.g. 92 ÷ 4 by taking away 20 lots of 4, to be left with 12, then taking away 4 x 3 to get the answer of 23.
- Using related facts to divide
- Using factor pairs to divide
- Scaling down using known facts
- Using the relationship between multiplication and division

Written Methods:

Written methods of division are not included as a National Curriculum objective in Year 4, however the non-statutory guidance states that children should develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.

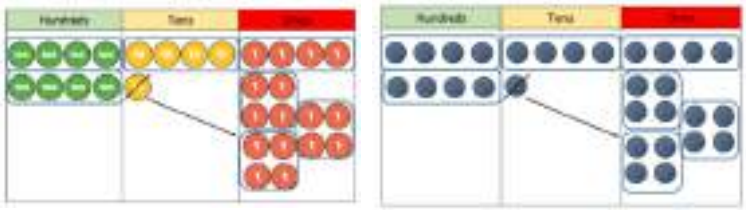
Short division with and without regrouping.

Consolidate year 3 work on division, using place value counters to support dividing a two-digit number by a one-digit number.

Children move onto dividing numbers with up to 3 digits by a one-digit number in a wide range of contexts.

Language is important: Children should consider 'How many groups of 4 hundreds can we make?'...

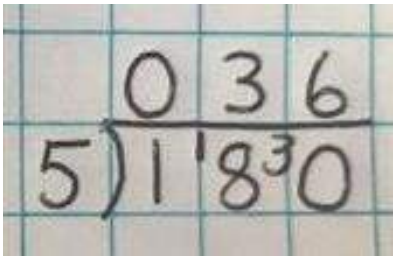
Children can draw their own counters and group them through a pictorial method.



		2	1	4
	4	8	5	$^{1}6$

Children should be familiar with the vocabulary **dividend, divisor** and **quotient** to support them in their understanding of the structure of division and so they are more able to explain their thinking and reasoning.

Where the answer to the first column is 0, children should initially write 0 above to acknowledge this and support with place value, then move the number over to the next column and regroup as a remainder.



Within problem solving, children should be encouraged to consider when to use a written method and when a mental method would be more efficient.

DIVISION

Year 5

National Curriculum Objectives: Division objectives from Multiplication and Division Strand	Key Skills/ other linked NC Objectives	Key Vocabulary
• Divide numbers mentally, drawing upon known facts. • Divide numbers up to 4 digits by a one-digit number using short division and interpret remainders appropriately for the context. • Divide whole number and those involving decimals by 10, 100 and 1,000. • Solve problems using division and a combination of the four operations.	• Identifying all factor pairs of a number and common factors of 2 numbers. • Know and use vocabulary of prime numbers, prime factors and composite (non-prime) numbers. • Establish whether a number up to 100 is prime and recall prime numbers up to 19.	Prime number, prime factors, common factor, composite (non-prime) number Dividend, divisor, quotient Divisible by, factor Group, groups of, lots of Equal, not equal Share, share equally Array Division, divided by, shared by, Grouping, sharing, left, left over

Children should:

- Be given the opportunity to continue to explore division in an increasingly wide range of real-life problems.
- They should consolidate and extend their use of short division, to include those calculations with remainders in their final answers.
- Significant time and teaching should be spend considering the meaning of those remainders and how they should be presented and interpreted, as this will enable children to have a more secure understanding in preparation for more complex problem solving in Year 6.

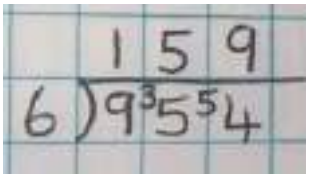
Mental Methods:

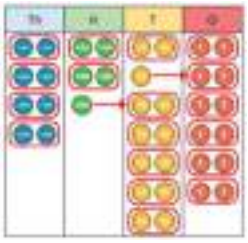
- Counting in steps of powers of 10.
- Recalling division facts for all the times tables, up to 12x12
- Use understanding of place value and what happens to the value of each digit when it is divided by 10, 100 or 1,000.
- Using known facts and place value to solve calculations.
- Using related facts to divide
- Using factor pairs to divide
- Scaling down using known facts
- Using knowledge of division facts e.g. when carrying out a division to find a remainder.
- Using the relationship between multiplication and division.

Written Methods:

Step 1: Dividing numbers with up to 4-digits by a one-digit number with no remainders in the final answer

Revise work from year 4 and then move onto dividing numbers with up to 3 digits by a one-digit number in a wide range of contexts.

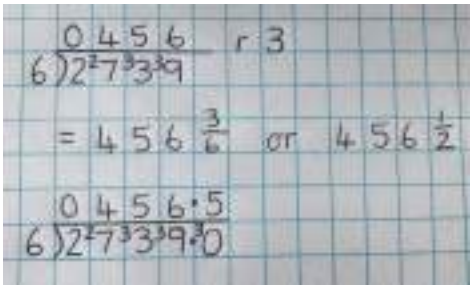




	4	2	6	6
2	8	5	$^{1}3$	$^{1}2$

Concrete apparatus can still be used to support, but children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

Step 2: Short division remainders Children should be exposed to examples that have remainders within the final answer. They should be given the opportunity, through specific teaching and modelling, to consider the meaning of the remainder and how it should be expressed (i.e. as a fraction, a decimal, or as a rounded number, depending on the context of the problem).



DIVISION

Year 6

National Curriculum Objectives: Division objectives from Multiplication and Division Strand	Key Skills/ other linked NC Objectives	Key Vocabulary
• Divide numbers up to 4 digits by a two-digit whole number using long division and interpret remainders as whole number remainders, fractions, or by rounding as appropriate for the context. • Divide numbers up to 4 digits by a two-digit whole number using short division where appropriate, interpreting remainders as appropriate to the context. • Perform mental calculations including with mixed operations and large numbers. • Use estimation to check answers to calculations. • Solve problems involving addition. • Use knowledge of order of operations to carry out calculations involving the four operations.	• Identify common factors and prime numbers. • Use estimation to check answers to calculations.	Mean average *Consolidate all previous vocabulary.*

Children should:

- Be given the opportunity to develop their division skills in a range of contexts, with a focus on presenting their remainders appropriately for the context.
- Learn to use long division to divide by two-digit numbers, and use these methods efficiently.

Mental Methods:

- Counting in steps of powers of 10.
- Recalling division facts for all the times tables, up to 12x12
- Using understanding of place value and what happens to the value of each digit when it is divided by 10, 100 or 1,000.
- Using known facts and place value to solve calculations.
- Using knowledge of division facts e.g. when carrying out a division to find a remainder.
- Using factor pairs to divide
- Using the relationship between multiplication and division
- Consolidating all previously taught strategies.

Written Methods:

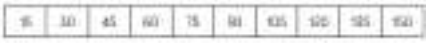
Extend use of short division for dividing by one-digit numbers.
Children continue to develop their use of short division and how to express remainders as whole numbers, fractions, rounded numbers and decimals. Specific teaching to take place to support children in understanding each of these and when they should be used.

Dividing by two-digit numbers

Short division

When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective.

Children should be encouraged to write out multiples to support their calculations.



Formal Method for Long Division

When the children have a clear understanding of the place value within their division calculations, they can move onto a formal method for long division. This reduces the amount of related facts that they need to use, and therefore will improve their efficiency.

2545 ÷ 12

How many groups of 12 hundreds are in 2 thousands? Zero – we need to regroup.

Regroup 2 thousands into 20 hundreds. We now have 25 hundreds.

How many groups of 12 go into 25 hundreds? 2 groups.

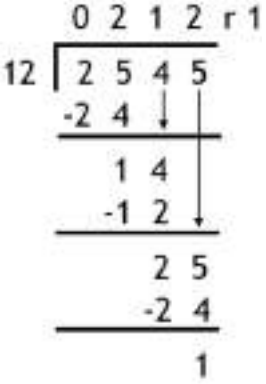
We have 1 hundred left to regroup into 10 tens. We have 14 tens.

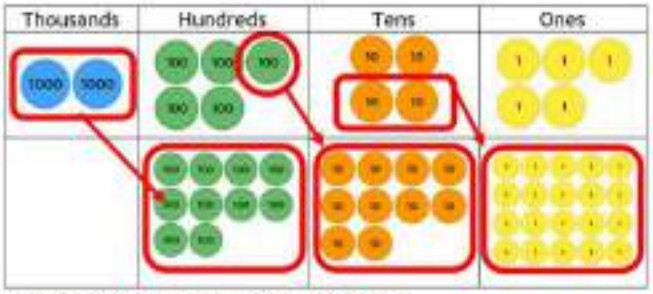
How many groups of 12 go into 14 tens? 1 group.

We have 2 tens left to regroup onto 20 ones. Now we have 25 ones.

How many groups of 12 go into 25 ones? 2 groups.

There is 1 one left over. This is our remainder: r 1.

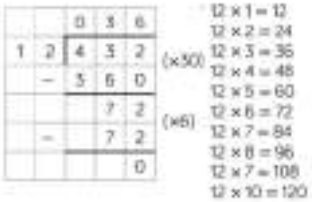




Therefore, 2545 ÷ 12 = 212 remainder 1.

Children can write out multiples to support their calculations.

432 ÷ 12 = 36



7,335 ÷ 15 = 489

When a remainder is left at the end of a calcualtion, children can either leave it as a remainder or convert it to a fraction or decimal. This will depend on the context of the question. Children will also answer questions where the remainder needs to be rounded depending on the context.

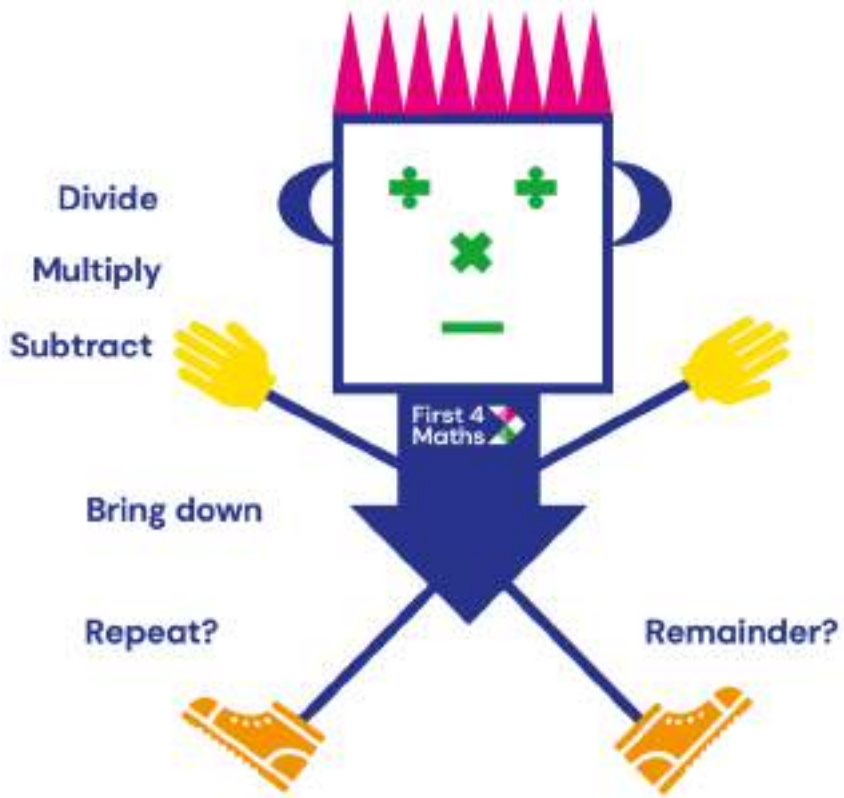
$372 \div 15 = 24\ r12$

$372 \div 15 = 24\frac{4}{5}$

			2	4	r	1	2
1	5	3	7	2			
	−	3	0	0			
			7	2			
	−		6	0			
			1	2			

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	−	3	0	0	
			7	2	
	−		6	0	
			1	2	



GLOSSARY

Addend	A number to be added to another.
Aggregation	Combining two or more quantities or measures to find a total.
Array	A physical or pictorial representation of a calculation, arranged in rows and columns, to help children understand multiplication and times tables
Associative law	The associative property says that when we add or multiply numbers it doesn't matter how we group them. E.g. 2 + (3 + 4) = (2 + 3) + 4 or (2 x 5) x 10 = 2 x (5 x 10)
Augmentation	Increasing a quantity or measure by another quantity.
Brackets	Symbols used to group numbers in arithmetic or letters and numbers in algebra and indicating certain operations as having priority.
Bridging	Bridging through 10 is a way of adding numbers greater than 10 in your head. For example, to add 8 + 7, you add 2 (from the 7) to get 10, then add the remaining 5 to get 15.
Bus Stop	The formal method for short division. It is a way of dividing numbers with two or more digits by one-digit numbers.
Calculate	To work out an answer using mathematical operations.
Calculation	A problem that can be solved using mathematical operations.
Column Method	The formal written method for completing addition and subtraction calculations. It can involve regrouping amounts from one column to the next (see regrouping).
Commutative	The word 'commutative' comes from 'commute' or 'move around', so **the commutative property refers to being able to move numbers around within number sentences. Addition and multiplication is commutative because n**umbers can be added or multiplied in any order.
Complement	The complement is the amount you must add to something to make it whole. In addition, a number and its complement make a total. E.g. 300 is the complement to 700 to make 1000.
Difference	The numerical difference between 2 numbers found by comparing the quantity in each group.
Distributive law	The rule that multiplying a number by a group of nu bers added together is the same as doing each multiplication separately. E.g. 17 x 3 = (10 x 3) + (7 x 3) or 2(5 +3) = 2 x 5 + 2 x 3.
Dividend	In division, the number that is being divided – the number inside the bus stop.
Division – sharing / grouping	Division can be **sharing** – the number to be divided is shared equally into the stated number of parts; or **grouping** – the number of groups of a given size is found. Division is the inverse operation to multiplication.
Divisor	The number you are dividing by - the number outside the bus stop (dividend ÷ divisor = quotient).
Equation	An equation says that two things are equal. It will have an equals sign. E.g. 2 + 3 = 5, 7 = 10 – 3, 3 + 5 = 10 – 2.
Grid Method	This a way for working out multiplication calculations, especially with larger numbers involving partitioning the numbers, multiplying each part and adding the totals (see partition).
Factor	A number that multiplies by another to make a product. E.g. 3 and 4 are factors of 12.
Flexible Partitioning	Splitting numbers into smaller numbers. Flexible partitioning can help children with mental calculations. E.g 8 + **7** = 8 + **2 + 5** or **96** ÷ 4 = **80** ÷ 4 + **16** ÷ 4)
Indices	Indices show how many times to multiply a number by itself. *(Also known as powers, roots or orders.)*
Inverse Operation	Another way of saying an opposite operation, which is used to check calculations are correct. Addition and subtraction are inverse operations, as are multiplication and division.
Long Division	The formal written method showing how to divide larger numbers (such as three or four digit numbers) by other large numbers. Children will move onto long division in UKS2, once they have mastered short division (see bus stop).
Long Multiplication	The formal written method for multiplying larger numbers together. The numbers are put in columns according to their place value.

Mean average	The size of each part when quantity is shared evenly.
Multiplicand	In multiplication, a number to be multiplied by another.
Multiplier	The number you are multiplying by. How many 'groups of'.
Multi-step Problem	Maths problems that require multiple calculations to solve them.
Minuend	A quantity or number from which another is subtracted.
Number Bonds	Pairs of numbers that add together to make a given number. For example, 2 + 8 and 4 + 6 are number bonds to 10, whereas 43 + 57 and 81 + 19 are number bonds to 100.
Partition	Splitting a number into its component parts based on place value (ones, tens, hundreds, thousands etc). 5,246 can be partitioned into 5 thousands, 2 hundreds, 4 tens and 6 ones or 5000 + 200 + 40 + 6
Partitive Division	Where we know the total number of groups and are trying to find the number of items in each group.
Place holder	Where zero is used to keep digits in the correct columns or determine the value of other digits.
Place Value	How much each digit in a number represents. In 157 the digit 5 has a value of 50.
Product	The result of multiplying one number by another. The product of 5 x 7 is 35.
Quotative Division	Dividing a given number into groups as a measured quantity, the goal is to find the number of groups.
Quotient	The result of a division. The amount each divisor receives (see divisor).
Reduction	Subtraction as takeaway.
Regrouping	To regroup means to change a number or expression for another of equal value. For example, we can regroup 1 ten into 10 ones, 1 thousand into 10 hundreds or 10 tens into 1 hundred.
Repeated Addition	A technique used to teach multiplication in Key Stage 1, where children add 'lots' of numbers together. For example, 3 'lots' of 5 is 5 + 5 + 5 as well as 3 x 5.
Remainder	The amount left over after a division when the divisor is not a factor of the dividend.
Scaling	Enlarging or reducing a number by a given amount, called the scale factor.
Subitise	Instantly recognise the number of objects in a small group without needing to count.
Subtrahend	A number to be subtracted from another. p
Sum	The result of an addition. The sum of 6 + 4 is 10
Total	The aggregate or the sum found by addition.